An heuristic market-based procedure for the collaborative assignment of ATFM resources

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***Abstract*—The allocation of ground delays to flights is a tactical tool commonly employed to control the flow of air traffic and to ensure that the available capacity of system resources is respected. Under the current European system these delays are assigned by the Network Manager according to a First-Planned-First-Served principle, without taking into account the individual cost of delay suffered by different flights. In this paper we formalize a decentralized, Individual Rational and Budget Balanced market-based mechanism for the collaborative assignment of delays among flights, based on the Lagrangian relaxation of the central assignment problem. It allows flights to pay for reducing their delays or to be compensated if they accept an increased delay with respect to the First-Planned-First-Served rule. We analyze its applicability under the general case of multiple capacity constrained resources and considering cost of delay functions for flights which are non-linear over time. The results obtained from a computational experience on a real traffic sample indicate that significant benefits could be achieved if it were implemented.**

***Index Terms*—Combinatorial exchange, Air Traffic Flow Management, Target Windows, User Driven Prioritization Process**

I. INTRODUCTION

The ATFM slot allocation measure is based on the universally accepted principle that delays on the ground are safer and less costly than those in the air. Thus a forecast delay somewhere in the system can be anticipated at the departure airport prior to the take-off and the traffic can be controlled in a safe and simple manner. These delays are assigned by EUROCONTROL Central Flow Management Unit (CFMU) according to a First-Planned-First-Served (FPFS) principle, on the basis of the Estimated aircraft Time Over the restricted location (ETO). The FPFS heuristic has longly been recognized fair by users, however it does not achieve in general the objective of cost minimization among concerned flights and significant benefits could be achieved if Aircraft Operators could take a more pro-active role in the management of ATFM delays (see e.g. [1])

SESAR will deeply modify this ATFM process through the introduction of the Business Trajectory, owned by the Airspace User, which evolves through several phases and represents the best trade-off between the constraints imposed by infrastructural and environmental restrictions and the users internal business objectives. Business Trajectories will be expressed in 4 dimensions (latitude, longitude, flight-level and time).

When an unexpected imbalance between capacity and demand is detected on a short notice, SESAR states that Airspace Users will be offered the possibility to indicate to the Network Manager a priority order for flights affected by delays under the so called User Driven Prioritisation Process (UDPP) [2].

In this paper we present a practical market-based mechanism to implement the collaborative assignment of resources whenever forecast demand is higher than system capacity, as it is the case for the UDPP. We extend the preliminary results presented in [3] by considering the general case of multiple constrained resources, which makes the underlying mathematical problem intractable and by formulating an heuristic algorithm that can be employed to find near-optimal solutions in an iterative way by directly querying airlines'preferences about different allocations.

A. Motivation for the study

A possible mechanism to formalize the SESAR Business Trajectory, comes from the concept of Contract of Objectives (CoO) which has been developed by the Contract-based Air Transportation System (CATS) research project (www.cats-fp6.aero). The CoO is a formal and collaborative commitment of ATM actors, i.e., Airspace Users, airports and Air Navigation Service Providers (ANSPs), to the conduction of each flight. It establishes a sequence of spatial and temporal constraints which constitute milestones to be met during the flight execution. These 4D intervals are called Target Windows and are agreed upon all involved actors for specific transfer of responsibility areas (e.g. between en-route control sectors). They represent the commitment to deliver a particular aircraft inside such temporal and spatial intervals. In other words, the proposed CoO consists of a collection of Target Windows defined at each area where responsibility between actors is transferred and the Business Trajectory should then go through these different Target Windows.

Since different airlines are in general competitors, market-based mechanisms seem natural ways to support a dynamic, collaborative and effective decision making process in the allocation of capacity.

In this context, we propose a market-based negotiation scheme between flights and the Network Manager to allocate Target Windows. The focus is on the 'critical' Target Windows which are those associated to the scarce air traffic resources, i.e., where the demand exceeds the available capacity. In fact, all the remaining Target Windows of each flight will be accordingly adjusted. For the sake of simplicity, in the remainder of the paper we only consider the temporal dimension of the critical Target Windows, thus in the following we simply refer to them as Time Windows (TWs).

Our work then relies on an extension of the current European ATFM system, under which just one departure ATFM slot may be allocated to a flight when the capacity does not meet the demand. Rather we assume that one TW is explicitly allocated to a flight for each congested resource crossed, in line with the 4-D trajectory management and the short-term capacity management principles proposed by SESAR. In particular, TWs are first allocated to flights following a First-Planned-First-Served rule at no cost, similarly to the current way of allocating ATFM slots. Then a market mechanism is initiated among flights coordinated by a Network Manager to negotiate available capacity, represented by a limited number of tradable TWs.

The rest of this paper unfolds as follows. Section II briefly indicate the most relevant literature and highlights the innovative contribution of the work. Section III illustrates the problem of allocating TWs to flights. Section IV formalizes this problem as a combinatorial exchange, introduces TW prices and discusses potentialities and limitations of the centralized approach. Section V proposes a distributed market mechanism to determine the optimal TW exchange and the associated TW prices. An heuristic algorithm is presented along with some computational results based on a real case instance. Section VI summarizes conclusions and highlights potential extensions to the study.

II. CONTRIBUTION OF THIS WORK

Following the seminal work of [4], a number of researchers have focused their activity on the development of optimization models and algorithms for the assignment of ground delays as a short-term measure to regulate traffic flows. The problem of assigning ground delays to a set of flights in order to minimize an aggregated cost function, given airport capacity constraints, is known as Ground Holding Problem (GHP).

In its basic version the GHP assumes that only one airport in the system is subject to capacity constraints which are imposed only on arrival flights. This problem is referred to as the Single Airport Ground Holding problem (SAGHP) and has been formulated for different cases (see e.g. [5], [6])

In the case a network of airports is considered and propagation of delay can occur between successive flights, the problem is referred to as the Multi Airport Ground Holding problem (MAGHP) (see [7]). Under this general case the problem is typically modeled through an integer program that increases computational burden with respect to the SAGHP.

A further extension of the MAGHP is the one that also includes constraints on the capacities of en-route sectors of airspace and determines optimal speed adjustments of aircraft, besides their release time into the network. This is known as Air Traffic Flow Management problem (TFMP) (see [8]). By adding the additional possibility of re-routing flights to avoid determined airspaces, the problem is known as Air Traffic Flow Management Re-routing problem (TFMRP) (see [9], [10]).

The common characteristic of all these models is the the presence of a unique central decision maker, the Network Manager, which is in charge of assigning individual delays to flights in order to minimize a global objective function. This obtained by aggregating the direct operating costs caused to all regulated flights by ATFM restrictions. This approach is consistent with the current mode of operations of the European ATFM system, but is no longer valid under the context of a collaborative layered planning as advocated by SESAR. In fact since Airspace Users are the owners of the Business Trajectories, they must be involved in any decision causing a modification of their original requests.

Hence we investigate the use of market mechanisms to formalize a practical procedure that allows Airspace Users to directly negotiate a collaborative solution to solve the TFMP.

Some papers have already proposed market mechanisms to assign limited resources in the context of air traffic. They are usually based on combinatorial auctions for the long-term strategic allocation of airport slots in the US (see [11], [12], [13]). However, an auction mechanism for air traffic resources is not in general appropriate, since some airlines would pay to get the same resources that they currently receive at no cost. To overcome this limitation, our mechanism is based on a combinatorial exchange as defined in [14] taking the baseline FPFS solution as an initial asset which can be traded by participants.

A market mechanism is *Individual Rational* (IR) if it guarantees that every participant will have a payoff equal or higher than the payoff obtained without taking part to it. In addition a market mechanism is *Budget Balanced* (BB) if it does not require an external subsidization to run properly. In particular, it is strongly BB, if the payments collected by all actors sum up to zero (i.e., it just redistributes money among participants) or weakly BB if the payments collected by the Network Manager can only be positive or equal to zero.

Our market mechanism is IR and weakly BB. Every airline as well as the Network Manager are guaranteed to have a non-negative payoff from the participation. Furthermore, it is distributed meaning that flights do not need to communicate to the Network Manager confidential information as their cost of the delay. Finally, our proposed mechanism falls within the Collaborative Decision Making (CDM) framework, as airlines have to continually interact with the Network Manager.

III. THE CENTRAL ALLOCATION PROBLEM

Let $\mathcal{F} = \{1, ..., F\}$ be a set of flights and $\mathcal{S} = \{1, ..., S\}$ a set of capacity constrained sectors and airports. Each flight $f \in \mathcal{F}$ is expected to cross a sequence of elements $S_f \subseteq \mathcal{S}$ according to its flight plan, hence it will need to be assigned a TW for each $s \in S_f$. A regulated resource $s \in \mathcal{S}$, with capacity limited to K_s entries per hour from st_time to end_time, has an associated TW Allocation List $L_s = [1, ..., N_s]$. Each TW $j = [I_j, U_j] \in L_s$ has capacity of one flight where:

$$\begin{aligned} N_s &= \left\lfloor \frac{end\_time - st\_time}{\frac{60}{K_s}} \right\rfloor \\ I_j &= \left\lfloor st\_time + (j-1)\cdot\frac{60}{K_s} \right\rfloor \;\text{ with } j \in \{2, ..., N_s\} \\ U_j &= I_{j+1} - 1 \;\text{ with } j \in \{1, .., N_s - 1\}, \end{aligned}$$

and $I_1 = st\_time$, $U_N = end\_time$. We assume that the Flight Plan also indicates an estimated time of entry into each element $s \in S_f$ traversed by flight f, i.e. E_f^s. Then f is allocated a list of TWs $q_f = [w_1, .., w_{|S_f|}]$, where w_i is the TW assigned on the i^{th} element of S_f and can not be earlier than E_f^i since flights cannot be anticipated, i.e. $E_f^i \leq U_{w_i}$ for all $w_i \in q_f$. Additionally whenever $|S_f| > 1$, we assume that the flying time between pairs of consecutive elements, (i, j) with $i, j \in S_f$ and $j = i + 1$, is fixed. An assignment q_f will cause a positive delay to flight f if and only if $E_f^i < I_{w_i}$ for some $w_i \in q_f$ and the amount of delay will be:

$$d_f^{q_f} = \begin{cases} \max\limits_{i \in S_f}(I_{w_i} - E_f^i) & \text{if } E_f^i \leq I_{w_i} \quad \forall i \in S_f \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

Hence each assignment of TWs q_f to a flight f implies a nonnegative cost of delay $C(f, q_f) \geq 0$, which depends on several factors, e.g, the type of aircraft, the number of connecting passengers, the time of day, the amount of delay[15]. Possibly, airlines only can correctly estimate the real costs of the delays of their flights, hence they have to be queried in a mechanism that seeks to minimize the cost of a TW allocation.

An assignment q_f is feasible for flight f if and only if (i) it contains one TW for each $i \in S_f$ and each pair (i, j) of consecutive TWs is connected by the fixed flying time $E_f^j - E_f^i$, (ii) it assigns a nonnegative delay to f and (iii) the delay it assigns is bounded, i.e. $d_f^q < MaxDel_f$ where $MaxDel_f$ is a fixed parameter for each flight beyond which the flight prefers to be canceled.

Let us indicate with Q_f the set of all assignments that are feasible for flight f, then the optimal assignment of TWs to flights can be formulated as the following 0-1 IP program:

$$Z_{IP} = \min \sum_{f \in \mathcal{F}} \sum_{q \in Q_f} C(f, q) x(f, q) \tag{2a}$$

s.t.

$$\sum_{f \in \mathcal{F}} \sum_{q \in Q_f : q \ni w} x(f, q) \leq 1 \quad \forall s \in \mathcal{S}, w \in L_s \tag{2b}$$

$$\sum_{q \in Q_f} x(f, q) = 1 \quad \forall f \in \mathcal{F} \tag{2c}$$

$$x(f, q) \in \{0, 1\} \quad \forall f \in \mathcal{F}, q \in Q_f. \tag{2d}$$

The objective is to find the minimal cost assignment such that each TW is allocated at most to one flight (constraints (2b)) and each flight is assigned one bundle of TWs among its feasible ones (constraints (2c)). The binary decision variable $x(f, q)$ will be one if flight f receives the q^{th} bundle in its request set Q_f, zero otherwise. A feasible solution will exist if and only if there are enough TWs and requests, such that the assigned bundles are pairwise disjoint, i.e. they do not share any TW. To guarantee the existence of a feasible solution we assume that each regulated resource $s \in \mathcal{S}$ has an infinite capacity after the termination of its regulation and that each flight has a request $q_l \in Q_f$ that includes only TWs after the termination of each regulation traversed. The cost $C(f, q_l)$ associated to this bundle will be equal to either the cost of delay caused by such a bundle or to the cost of cancellation in the case this delay exceeds $MaxDel_f$.

Hence to achieve the optimal TW assignment the Network Manager could centrally solve Problem 2, with weights $C(f, q)$ communicated by Aircraft Operators, to find an assignment $X^* = \{q_1^*, ..., q_F^*\}$ that minimize the total cost of delay. However in this case there would obviously be an incentive for users to declare weights $\widehat{C}(f, q) > C(f, q)$, higher than true value in order to receive a less penalizing assignment. To avoid this issue Aircraft Operators could be charged an appropriate price $p(q_f^*)$ for the bundle of TWs k^* assigned by solving Problem 2 with weights $\widehat{C}(f, q)$ declared by users. Then if we assume, as in standard auction theory, that each flight has a quasi-linear utility given by $u(f, q_f^*) = -C(f, q_f^*) - p(q_f^*)$, the auction is not IR, since $u(f, q_f^*) \leq 0$ for all $f \in \mathcal{F}$.

To force IR we rather propose to calculate the standard FPFS assignment $A = \{a_1, ..., a_F\}$ as in the current system and to consider it as the initial endowment for each flight. This is a feasible solution for Problem 2 but not necessarily optimal, then we propose to implement the exchange between A and X^* such that every flight f for which $a_f \neq q_f^*$ pays the price $p(q_f^*)$ for its optimal bundle but also receives the payment $p(a_f)$ for the released bundle a_f. Then each flight is a potential buyer and seller of TWs and its utility after the exchange will be $u(f, e^*) = [C(f, a_f) - C(f, q_f^*)] - [p(q_f^*) - p(a_f)]$.

IV. PRICING THE EXCHANGE

Through a linear transformation the problem of finding the minimum cost TW allocation (Problem 2) can be reformulated as the problem of finding the maximal value TW exchange through the following IP model:

$$Z_{IP-E} = \max \sum_{f \in \mathcal{F}} \sum_{q \in Q_f} V(f, q) x(f, q) \tag{3a}$$

s.t.

$$\sum_{f \in \mathcal{F}} \sum_{q \in Q_f : q \ni w} x(f, q) \leq 1 \quad \forall s \in \mathcal{S}, w \in L_s \tag{3b}$$

$$\sum_{q \in Q_f} x(f, q) = 1 \quad \forall f \in \mathcal{F} \tag{3c}$$

$$x(f, q) \in \{0, 1\} \quad \forall f \in \mathcal{F}, q \in Q_f \tag{3d}$$

where $V(f, q) = [C(f, a_f) - C(f, q)]$ is the value obtained by flight f by exchanging bundle a_f with bundle q and will be positive if the delay caused by bundle q is lower than delay caused by a_f, negative otherwise. We want to find prices such that for each flight f, $u(f,e^*) \geq 0$ in order to guarantee IR and $\sum_{f\in\mathcal{F}} p(e_f^*) \geq 0$ in order to guarantee (weak) BB.

We define Z_{LP-E} as the objective value of the linear relaxation of Problem 3, whose dual problem is:

$$Z_{DLP-E} = \min \sum_{f\in\mathcal{F}} u_f + \sum_{s\in\mathcal{S}'} \sum_{w\in L_s} p(w) \tag{4a}$$

s.t.

$$u_f + \sum_{s\in\mathcal{S}'} \sum_{w\in(L_s\cap q)} p(w) \geq V(f,q) \quad \forall f\in\mathcal{F}, q\in Q'_f \tag{4b}$$

$$p(w) \geq 0 \quad \forall s\in\mathcal{S}', w\in L_s \tag{4c}$$

The formulation of Problem 4 suggests the interpretation of dual variables as supporting prices $p(w)$ for TWs. Let us assume a linear structure of prices, i.e. for of each bundle of TWs $q\in Q_f$ its price will be $p(q) = \sum_{w\in q} p(w)$.

The complementary slackness conditions for the primal-dual pair imply that a linear pricing rule based on individual prices $p(w)$ implements an individual rational mechanism for our TW-exchange problem, whenever the LP relaxation of Problem 3 gives an integer solution. In fact each flight f will weakly prefer to sell the bundle of TWs $a_f \in Q_f$ received under FPFS allocation if it is not optimal for Problem 2a and to buy the bundle of TWs $q_f^* \in Q_f$ optimal for Problem 3, because its utility increases:

$$C(f,a_f) + \sum_{w\in a_f} p(w) - C(f,q_f^*) - \sum_{w\in q_f^*} p(w) \geq 0 \tag{5}$$

Such a payment rule is (weakly) BB, since it can produce a monetary surplus for the Network Manager. In fact all TWs unassigned under FPFS which are assigned in the optimal allocation have a price $p(w) > 0$ according to complementary slackness conditions. This price has to be paid by the receiving flight but is not due to anyone since that TW was unallocated under FPFS. On the contrary any TW that is not assigned in the optimal allocation has a price $p(w) = 0$, meaning that the flight giving up a TW received under FPFS which is not allocated by the market mechanism does not receive any compensation for it.

Also the solution constitutes a *competitive equilibrium*, i.e. at current prices (also called market-clearing or supporting prices) no participant can be better off by performing a different exchange.

Unfortunately, supporting prices corresponding to the solutions to Problem 4 exist only in the case that the integrality gap between Problem 3 and its LP relaxation is null. This will always be verified in the case of gross-substitute valuation functions [16], i.e., if for every pair of price vectors $p' \geq p$ (component-wise comparison), we have that the optimal TW package demanded by a flight f at prices p' contains all the TWs in the optimal package demanded by f at prices p, whose price remained constant.

The class of gross-substitutes is a largest set of valuation functions which contains unit-demand ones, i.e. when all flights demand bundles are composed by a single TW, or equivalently in the case of a single constrained resource. In those situations Problem 3 reduces to an assignment problem, thus implying that its linear relaxation gives integer feasible solutions, and dual variables define linear market-clearing prices (see [17]). Whenever there are complementarities in valuations functions (i.e. in the case of multiple constrained resources), gross-substitutes property does not hold anymore.

V. A DISTRIBUTED MARKET MECHANISM

The central allocation problem that determines the optimal exchange, could be complicate to adopt in practice, principally because (i) it requires the complete disclosure of airline private information regarding their cost and (ii) the computational burden for solving Problem 3 is entirely faced by the Network Manager which must solve one NP-hard problem for each allocation (in fact Problem 3 is NP-hard by reduction to the Weighted Set Packing Problem). In order to avoid these issues we propose in the following a distributed algorithm that exploits the decomposition properties of Problem 3. In fact, by dualizing constraints (3b) the corresponding Lagrangian formulation of Problem 3 is:

$$ZLR_{LP-E}(\lambda) = \max \sum_{f\in\mathcal{F}} \sum_{q\in Q_f} V(f,q)x(f,q) + \\ + \sum_{s\in\mathcal{S}, w\in L_s} \lambda_w \Big(1 - \sum_{f\in\mathcal{F}, q\in Q_f : q\ni w} x(f,q)\Big) \tag{6a}$$

s.t.

$$\sum_{q\in Q_f} x(f,q) = 1 \quad \forall f\in\mathcal{F} \tag{6b}$$

$$x(f,q) \geq 0 \quad \forall f\in\mathcal{F}, q\in Q_f \tag{6c}$$

Problem 6 is separable into F problems, one for each flight and can be solved locally by Aircraft Operators, according to Problem 7, which is a linear problem and can thus be solved in polynomial time:

$$ZLR_{LP-E}(f,\lambda) = \max \sum_{q\in Q_f} V(f,q)x(f,q) + \\ + \sum_{s\in\mathcal{S}, w\in L_s} \lambda_w \Big(1 - \sum_{q\in Q_f : q\ni w} x(f,q)\Big) \tag{7a}$$

s.t.

$$\sum_{q\in Q_f} x(f,q) = 1 \tag{7b}$$

$$x(f,q) \geq 0 \quad \forall q\in Q_f \tag{7c}$$

For each TW w, its prices $\lambda(w)$ are calculated centrally according to the excess of demand for it and then communicated to Aircraft Operators, which will in turn modify the demand for TWs according to such prices. The following algorithm can be employed to calculate prices

$$\lambda_w^{k+1} = \max(0, \lambda_w^k - S_r^k \cdot SG_s^k) \tag{8a}$$

$$SG_s^k = 1 - \sum_{f \in \mathcal{F}, j \in Q_f : q \ni w} x(f,j) \tag{8b}$$

where S_r^k is a positive stepsize chosen at iteration k and SG_s^k is a subgradient of $ZLR_{LP-E}(\lambda)$ at any λ for which x solves Problem 6. Thus ideally the Network Manager seeks the prices λ that solve the following dual problem

$$ZLR_{LP-E} = \min_{\lambda} ZLR_{LP-E}(\lambda) \tag{9a}$$

s.t.

$$\lambda \geq 0 \tag{9b}$$

Since $ZLR_{LP-E}(\lambda)$ is a convex, piecewise linear, non-differentiable function, this problem is typically solved through a subgradient algorithm. The resulting procedure iteratively alternates a central price-calculation phase (Problem 8) with a local optimization one which finds the maximal value TW-exchange at current prices (Problem 7). By appropriately choosing the stepsize S_r^k such that $S_r^k \to 0$ and $\sum_{i=1}^k S_r^k \to \infty$ for $k \to \infty$, the procedure converges to a solution which minimizes $ZLR_{LP-E}(\lambda)$[18]. By duality theory it will follow that $Z_{IP-E} \leq Z_{LP-E} \leq ZLR_{LP-E}(\lambda)$, while $Z_{IP-E} = Z_{LP-E}$ in the case of null gap between the integer program and its linear relaxation (i.e. with gross-substitutability) and $Z_{LP-E} = ZLR_{LP-E} = \min_\lambda ZLR_{LP-E}(\lambda)$ when the subgradient algorithm converges to an optimal solution for Problem 9.

However this exchange will be optimal for the original problem if and only if the gap between Z_{IP-E} and its linear relaxation is null, a condition which can only be guaranteed in the case of gross-substitute valuations, for example when all participants compete for TWs on a unique resource. Furthermore, even in the case of gross-substitutability, there is no guarantee of convergence in a finite number of steps. By stopping the procedure when a feasible exchange for the original Problem 3 is demanded at current prices, the optimality of the solution will be verified if and only if $\sum_{s \in \mathcal{S}, w \in L_s} \lambda_w (1 - \sum_{f \in \mathcal{F}, q \in Q_f : q \ni w} x(f,q)) = 0$ [19]. Hence if the procedure stops prematurely it will be $ZLR_{LP-E} > Z_{IP-E}$ and some lagrangian multipliers λ_w will be higher than minimal prices. Then the solution would not constitute a competitive equilibrium, but the correspondent exchange will still guarantee IR and weak BB. We propose in the following section an heuristic algorithm to implement the distributed exchange that exploits some of these properties to realize a practical mechanism for TW exchanges.

A. A heuristic approach

In order to implement a distributed market mechanism that achieves, in a reasonable amount of time, an exchange which is IR and BB we propose the following heuristic. A formula for S_r^k which has been proven effective in practice is:

$$S_r^k = \frac{\mu_k (ZLR_{LP-E}(\lambda^k) - Z^*_{IP-E})}{\sum_{s \in \mathcal{S}, w \in L_s} (1 - \sum_{f \in \mathcal{F}, q \in Q_f : q \ni w} x^k(f,q))^2} \tag{10}$$

where $0 < \mu_k \leq 2$, x^k are the solutions to Problem 7 at iteration k according to the vector of TW prices λ^k. Usually the scalar μ_k is taken at its higher values during first iterations and halved whenever $ZLR_{LP-E}(\lambda^k)$ has failed to decrease in a specified number of iterations[20]. In our case the Network Manager does not know the exchange values $V(f,q)$ and thus cannot calculate neither $ZLR_{LP-E}(\lambda^k)$ nor Z^*_{IP-E}. We then modify formula (10) in the following sense:

$$S_r^k = \frac{\mu_k (UB\_Z^* - ZLB_{IP-E})}{\sum_{s \in \mathcal{S}, w \in L_s} (1 - \sum_{f \in \mathcal{F}, q \in Q_f : q \ni w} x^k(f,q))^2} \tag{11}$$

where UB_Z^* is an upper bound on the optimal value of the exchange which is held constant and ZLB_{IP-E} is a lower bound on the optimal value of the exchange for each instance, which is dynamically adjusted through the course of the distributed mechanism. At iteration k the Network Manager can in fact calculate for each bundle j demanded by flight f at current prices λ^k, a lower bound on the exact value $V(f,j)$ for the exchange:

$$LB(f,j) = \sum_{s \in \mathcal{S}, w \in L_s : j \ni w} \lambda_w^k - \sum_{s \in \mathcal{S}, w \in L_s : a_f \ni w} \lambda_w^k \tag{12}$$

For all $f \in \mathcal{F}$ and $q \in Q_f$, lower bound can be initialized to $LB(f,q) = 0$ if $d_f^q \leq d_f^{a_f}$ and $LB(f,q) = -\infty$ if $d_f^q > d_f^{a_f}$, since we assume that the cost of delay is a non-decreasing function of the duration. This implies that each flight would exchange its FPFS assigned bundle a_f with q for a cost greater or equal to zero whenever q causes a shorter delay then a_f or for a negative cost (a taking) whenever q represents a longer delay then a_f. At iteration k the Network Manager will calculate the $LB(f,j)$ value according to formula (12) and it will store it in memory if it is higher than the previously calculated one. Also it is possible to update with this same value the $LB(f,b)$ for all the bundles $b \in Q_f$ such that $d_f^b < d_f^j$, since due to monotonicity of the cost functions it will be $V(f,b) > V(f,j)$. The Network Manager can then solve the following problem:

$$ZLB_{IP-E} = \max \sum_{f \in \mathcal{F}} \sum_{q \in Q_f} LB(f,q) x(f,q) \tag{13a}$$

s.t.

$$\sum_{f \in \mathcal{F}} \sum_{q \in Q_f : q \ni w} x(f,q) \leq 1 \quad \forall s \in \mathcal{S}, w \in L_s \tag{13b}$$

$$\sum_{q \in Q_f} x(f,q) = 1 \quad \forall f \in \mathcal{F} \tag{13c}$$

$$x(f,q) \in \{0,1\} \quad \forall f \in \mathcal{F}, q \in Q_f \tag{13d}$$

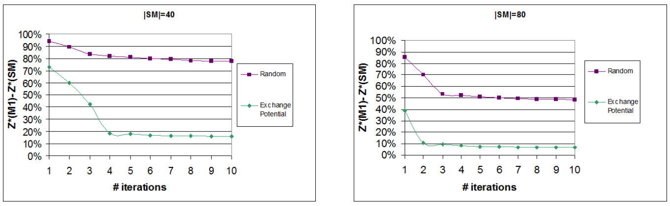
Problem 13 is equivalent to Problem 3 with $V(f,q) = LB(f,q)$ for all $f \in \mathcal{F}$ and $q \in Q_f$, then it will be $ZLB_{IP-E} \leq Z_{IP-E}$, the strict inequality holding whenever $LB(f,q) < V(f,q)$ for at least one request $q \in Q_f$ for some flight $f \in \mathcal{F}$. Hence all the integer (feasible) exchanges calculated by solving the linear relaxation of Problem 13 with $V(f,q) = LB(f,q)$ and implemented at prices equal to the dual variables corresponding to constraints (13b), will guarantee IR and weak BB, whenever $ZLB_{IP-E} > 0$.

The solution obtained (exchanges and prices associated) is not however a competitive equilibrium because at the given prices there could be some f better-off with another exchange than the one implemented and this implies that the solution must be somewhat forced by the Network Manager. Then after a predetermined number of iterations or a given elapsed time, if an equilibrium cannot be found by simply alternating local optimization and price update (i.e. Problems 9 and 8), then the Network Manager can impose the best solution calculated so far by solving the linear relaxation of Problem 13 at the dual prices, i.e. the integer solution which gives the highest positive-value according to LB, which is the last feasible solution obtained since LB are always updated by increasing them.

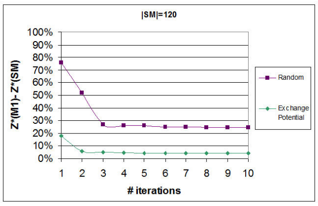
In order to speed-up the convergence to a solution the procedure creates a partition of the grand coalition $\mathcal{F}$ into independent subsets $M_i \subseteq \mathcal{F}$, such that for every pair of different flights $f \in M_i$ and $g \in M_j$ with $i \neq j$ it will be $Q_f \cap Q_g = \emptyset$. Then each subset M_i constitutes an independent market, since all the tradable resources will be within the market itself.

From each independent market, smaller sub-coalitions (sub-markets) of predetermined size Z are formed and then processed, in order to ease the coordination of the involved participants by reducing their number and at the same time to increase the probability of obtaining integer solutions to the linear relaxation of Problem 13. In fact we have observed experimentally that by reducing the size of the sub-markets, at the same time the value of the optimal exchange reduces while the percentage of instances for which $Z_{IP-E} = Z_{LP-E}$ increases.

Sub-markets are created according to their exchange potential. Flights in Market M_i are first ordered from the one with the lowest to the one with the highest assigned FPFS request. Then starting from the head of this ordered list one flight f is selected as well as its first potential seller g starting from the tail. A flight g is a potential seller for flight f if (i) they share at least one resource s ($S_f \cap S_g \neq \emptyset$) (ii) f prefers the TW k assigned to g on s than its currently assigned one j ($I_k < I_j$) (iii) TW k is feasible for f ($E_f^s \leq U_k$). If no potential seller exists the flight next to f is selected together with its first potential seller. Once a number of flights of the predetermined size Z has been selected, the first sub-market is created. Then a second sub-market which considers only flights which have not been previously included is built. This procedure may iteratively create up to $|M_i|/Z$ distinct sub-markets of size Z.



(a) $|SM| = 40$ (b) $|SM| = 80$



(c) $|SM| = 120$

Fig. 1. Gap between the optimal value of the exchange in the Main Market $M1$ and in Sub-markets SM.

Relaying on the market M_1 of 425 flights as described in Section V-B, Figure 1 illustrates the case in which 10 sub-markets of fixed size $|SM|$ successively trade TWs in order of their potential of exchange, in comparison with the situation in which sub-markets are formed by including flight at random. Each point corresponds to the average on 100 instances of different vectors of costs drawn from the same distribution.

By processing flights according to successive sub-markets one can attain up to 97% (in the case of $|SM| = 120$) of the optimal value obtainable by processing the entire main market and the exchanges with highest value occur in the first 3 or 4 sub-markets depending on the size $|SM|$. The solutions obtained always dominate the case in which sub-markets are formed randomly from the main market.

The number of instances which give non integer solution to the linear relaxation of Problem 3 increases with the size of the sub-market. When $|SM| = 40$ on average 0.15% of cases are non integer, 3.65% when $|SM| = 80$ and 4.35% when $|SM| = 120$. In these cases the dual variables are not supporting prices and one possible solution could be represented by the exchange optimal for the integer Problem 3 and the prices calculated according to a VCG-based payment rule. Even if a competitive equilibrium with linear prices does not exist for such instances, our heuristic can still converge to a solution which guarantees IR and weak BB. Once the sub-markets have been formed according to criteria described before, the iterative mechanism can be applied to them.

If after a predetermined number of iterations $MaxIter$ an equilibrium cannot be found, the last feasible solution calculated with LB is imposed by the Network Manager and the correspondent exchange is implemented at the dual prices. The diagram in Figure (2) represents the steps performed by the heuristic procedure.

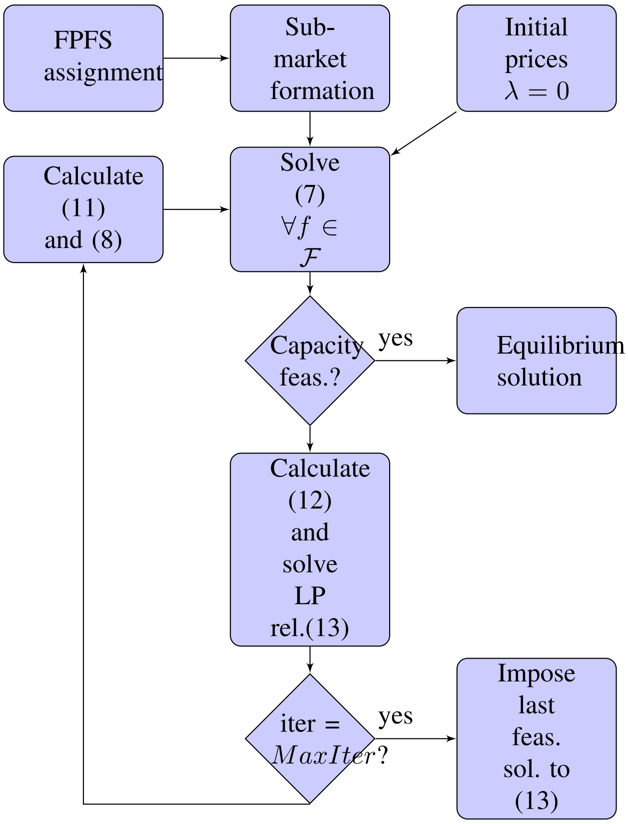


Fig. 2. Schematics of the iterative Market Mechanism

B. Computational results

We simulated this procedure on a sample of traffic retrieved from real CFMU data relative to the two hours period from 09:00 AM to 11:00 AM on Friday August 15^{th}, 2008. There were a total of 60 capacity constrained resources and 482 regulated flights, that were clustered into 3 independent markets $(M1, ..., M3)$, with $|M1| = 425$,$|M2| = 34$,$|M3| = 23$. Market $M1$ included most of the flights interacting directly or indirectly in the exchange of TWs on 58 resources, flights in $M3$ and only them were affected by a single regulation on an en-route sector, while Flights in $M2$ and only them planned to cross a sector that was closed from 10.15 AM to 11.30 AM due to ATC routing. We then focused the simulation on the main Market M1.

We model the cost of delay for each flight $f \in \mathcal{F}$ as a piecewise-linear function of its duration through the vector $CD_f \subset \mathbb{N}^3$, where each component represents the per-minute cost of delay according to the magnitude of the delay itself, which has been discretized into the three classes $[1; 15)$ min, $[15; 45)$ min, $[45; MaxDel_f]$ min, in line with the figures provided by [15]. Components $cd_f \in CD_f$ have been randomly drawn from the uniform distribution on the three discrete intervals $[1; 5]$ €/min, $[15; 25]$ €/min, $[30; 105]$ €/min respectively. We fixed $MaxIter = 150$. Graph in Figure (3) shows the results obtained by applying the iterative Market Mechanism presented schematically in Figure (2).

TABLE I
COST OF DELAY WITH DIFFERENT METHODS FOR TW ASSIGNMENT (ALL FIGURES IN €).

Central assignment		Iterative mechanism			
FPFS	Central optimal	with sub-markets			without sub-markets
		$\|SM\|$=40	$\|SM\|$=80	$\|SM\|$=120	$\|SM\|$=425
740187	620692	661101	640910	634438	629331

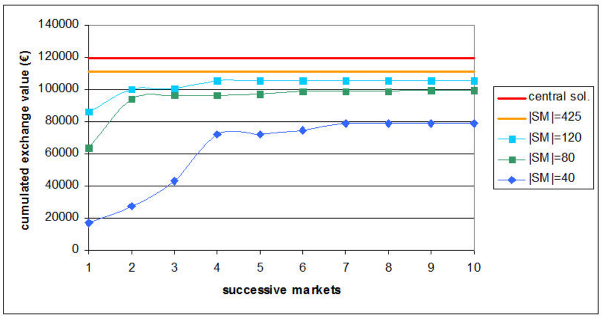


Fig. 3. Value of the exchanges obtained through the iterative mechanism on several Sub-Markets

Table I summarizes the main aggregated high level figures on delay cost, resulting after applying different TW allocation policies. We employed Xpress Mosel v.3.0.0 to code the heuristic procedure and Xpress Optimizer version 20.00.05 to solve all the linear problems.

The first column shows the total cost of delay (€740187) due to the 12989 min of ground delay assigned to flights according to our algorithm implementing the FPFS allocation. This figure cannot match exactly the real one computed by the EUROCONTROL Central Flow Management Unit, due to the differences in both the behaviors of the algorithms and the environment of application, which is highly dynamical and influenced by many external factors (e.g. cancellations, reroutings, ecc.) in the real world. However this result is perfectly in line with the figures estimated by [21], which takes an average cost of delay of 63 €/min equal for all flights, while the average cost of delay produced by our simulation is 57 €/min.

The second column represents the total cost of delay (€620692) obtained by the central TW assignment following the combinatorial exchange (Problem 3). With respect to the total cost of delay originally caused by the FPFS allocation, we achieve cost savings approximately equal to the 20%.

The remaining four columns show the results obtained by applying the iterative Market Mechanism described in Section V-A.

The highest value is achieved by letting all flights $f \in M_1$ participate in a unique repetition of the mechanism (i.e. $SM = M_1$), however the coordination tasks for the Network Manager may become slower. In fact at each iteration k all the optimal demands at current prices λ^k have to be collected and they are likely to arrive asynchronously, depending on the computational capabilities of the aircraft operator. Additionally only a small percentage of solutions obtained by iteratively solve LP relaxation of Problem 13 will give feasible integer solutions (26% according to our simulations). This reduces the probability of finding linear prices that support the exchange.

In all the other three cases with $SM \subset M1$ the most valuable exchanges occur within the first five repetitions of the procedure on successive sub-markets. For the case with $|SM| = 40$, 90% of the final value is attained after 4 trades series, while with $|SM| = 80$ and $|SM| = 120$ just 2 repetitions of the procedure are sufficient to achieve 95% of the final value. This is due to the higher number of exchanges which becomes possible by increasing the sizes of coalitions and it proves that remarkable cost savings are achievable also by employing a decentralized heuristic negotiation mechanism.

We estimate that by employing a fast communication network as the one provided by SWIM and by automating the airline interfaces through a proxy for the computation of demands, each iteration will require a time in the order of 5 to 10 seconds, then in less than 30 minutes the most valuable exchanges could be calculated and implemented.

VI. CONCLUSIONS AND FUTURE WORK

The problem of collaborative assignment of ATFM resources based on the TW concept proposed by the CATS project was analyzed. Several interesting properties of its mathematical structure were highlighted and a practical mechanism that can be adopted at a pre-tactical phase was formalized. The computational experience executed on a real traffic sample shows that great benefits are achievable if Airspace Users could trade TWs seeking to minimize their cost functions.

This work could be extended in several directions. First it would be interesting to study the case of Traffic Flow Management Rerouting Problem, where each flight can follow several alternative possible routes corresponding to different preference rankings and allowing variable velocity during flight to meet the assigned TW; another feature could be the introduction of TWs of variable duration, whose price can modify depending on their time extension, providing an incentive to Airspace Users to narrow down their time predictions.

Standard currency for implementing monetary payments was only considered in this study, however it would be worth analyzing the introduction of a dedicated monetary system and its impacts on the implementation of the mechanism, in particular on Incentive Compatibility. Also it would be interesting to simulate the application of the mechanism on a continuous rolling-horizon basis, in which flights could enter a few hours before take-off and leave either when a satisfactory exchange occurs or at a fixed time before its originally assigned TW. Feedback from different representatives of Airspace Users would be fundamental during the definition of the advanced mechanism and during its validation. This could imply expert groups and gaming exercises involving participants from different airlines.

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