Granting flexible operations in congested airspaces

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***Abstract*—Several causes of delay are deterring the air transportation system from being efficient. System capacity reductions are the major cause of delay. However, there are also some other causes, directly imputable to airlines, which may produce, in combination with the capacity reductions, undesired downstream effects. Therefore, with the purpose of containing delays and disruptions in flight schedules, it is important to grant flexibility to flight operations. This paper presents a mathematical formulation that allows to determine the degree of flexibility given to flights by identifying through a set of temporal intervals, called time windows, those flights that have a larger impact on the air traffic system performances. A time window is a period of time during which a certain phase of the flight (e.g., take off, landing and entry into a sector) has to be executed. The size of the time windows is variable as it reflects system's capacity constraints. The set of time windows, which maximizes the total width of the time windows, provides airline operators and air traffic control authorities with the largest degree of flexibility to perform their operations. Several formulations of the models are presented, which vary in the way of formulating the use of system capacity. However, by means of a computational analysis, we show that the solution of the time window model is insensitive to the formulation used for the capacity constraints.**

Keywords - Air Traffic Flow and Capacity Management, Time Windows, Delay, Critical Flights, ATFM, ATFCM

I. INTRODUCTION

Statistics on both the number of passengers and the quantity of goods carried are showing positive trends. This 'rosy' picture might be in contrast with the current situation of the air traffic system. In fact, the air traffic system is already facing high capacity constraints due to the limited availability of resources both on the ground and in en-route airspace [1,2] leading to imbalances between demand and capacity at key times and points of the air transportation network. The local congestion create delays which propagate to other parts of the air network, amplifying congestion as an increasing number of local capacity constraints come into play. In 2009, in Europe the share of arrivals delayed by more than 15 minutes was equal to 17,9% [3].

About half of departure delay are produced under airline control (e.g. maintenance or crew problems, aircraft cleaning, baggage loading, fueling, etc.) [3], thus confirming again the complex mix of resources necessary for the execution of a flight. It follows that a continuous negotiation process among air traffic controllers, airport operators and flight crews (or dispatchers) takes place and is detailed in an approved flight plan. The operator of a flight is expected to adhere as precisely as possible to the flight plan, although some adjustments are possible. However, there are flights which have to be operated in strict accordance to the approved flight plan, since any delay assigned to them may have large downstream effects such as disruptions in the airline schedules or degradations of the Air Traffic Control (ATC) system performances. For these flights, there is no slack time in handling their operations and a limited number of recovery options is generally available. On the other side, as some portions of the airspace may be less congested than others, a larger flexibility can be given to airspace users, air navigation service providers and airports operating in such sparse areas, without degrading the overall performance (i.e., the total cost of delay) of the entire system. In what follows we will call those flights with strict limitations on their operations as 'critical'.

Herein, we present a mathematical model to detect critical flights. The proposed formulation is based on the concept of 'time window', that is, a period of time during which a certain phase/operation of the flight (e.g., taking off, landing and entering sectors) has to be executed. Each time window is univocally determined by its position, i.e., starting time, and by its width. The width of the window delineates the degree of flexibility to carry out the operation. Indeed, the larger is the time window, the larger is the amount of slack time available to execute the flight operation. The smaller is the time window, the more critical is the flight. In this case, it is important that all the activities executed in support of flight operations, e.g. maintenance, ground and flight crew activities, and ATC clearances, are coordinated and executed on time. Hence, the execution of a flight within the assigned time windows shelters the flight operator/airline from any possible additional ATC delay. But, if the flight is not able to meet its time windows, i.e., some of its operations are executed after the end of the corresponding time windows, it may incur in an additional delay imposed by the ATC authorities because of air traffic congestion.

Since the smaller is a time window, the more critical is a flight, the proposed mathematical model detects critical flights by maximizing the total width of the time windows.

From the modeling point of view, a time window is composed of a discrete and limited number of time units, each representing a fixed time interval. As an example, if a time unit corresponds to a 5 minute interval, a time window composed of three time units is 15 minutes wide. As mentioned above, the time window is univocally determined by its starting time and its width. In non congested circumstances, the starting time may coincide with the time reported in the flight plan. However, if a congestion phenomena occurs, due to a lack of capacity, a shift of certain number of time windows might be necessary - thus producing ATC delays - to guarantee the satisfaction of safety requirements. An optimal position of the time windows, which minimizes the total amount of the initial ATC delay (shift), can be computed with any air traffic flow management (ATFM) model, see Odoni [4] for a first formalization and Hoffman et al. [5] for a recent survey on the topic.

The time window concept is consistent with the Single European Sky ATM Research (SESAR) program as it enhances the responsibility of airlines in the context of the Air Traffic Management (ATM) system [6]. In fact, airlines are driven to manage their operations in order to deliver their flights inside these temporal intervals, thus leading to improved planning and earlier detection of delays. In this context, the Contact-based Air Transportation System (CATS) research project (www.cats-fp6.aero) introduces the Contact of Objectives as a formal commitment among airspace users, airport and air navigation service providers for the conduction of each flight. The Contact of Objectives consists of a sequence of spatial and temporal constraints which constitute milestone to be met during the flight execution. These 4D intervals are called Target Windows inside which each air traffic actor engages in delivering its services to flight execution, from gate to gate [7]. A qualitative analysis of the benefits and drawbacks that the implementation of such windows may have on airspace users, airports and air navigation service providers both at the flight planning and flight execution levels is available in [8].

This paper unfolds as follows: Section II presents the mathematical formulation of the model to identify critical flights. Section III describes the computational experience to date and analyses the results. Finally, Section IV summarizes conclusions and indicates the next research steps.

II. Mathematical Model

The mathematical model described here is designed to compute time windows for each flight. We assume that the position (i.e., the starting time) of each time window is given by the optimal solution of an ATFM model. More precisely, we used the model presented in [9] whose solution – which minimizes the total cost of delay - describes the 4D trajectory of each flight *f*, by identifying the time period ti_j^f of arrival at airspace element *j* (airport or sector). Therefore, time period ti_j^f is the period when the time window for a flight *f* to operate in airspace element *j* opens. Hence, the model has to determine the optimal closing time for all time windows such that their total width is maximized. The mathematical model is also based on the following assumptions:

1. a time window is composed of a discrete and limited number of contiguous time periods of fixed width;
2. each phase of a flight is executed in exactly one time period.

In what follows, we introduce the notation, the decision variables and their the range of definition, the objective function and the constraints of the time window model.

A. Decision variables.

For each flight *f* and airspace element *j*, the opening time of the time window is set equal to ti_j^f provided by the optimal solution of the ATFM problem. Thus we only need to fix the right-end of the time windows, i.e., the decision variables are defined only for time periods subsequent to ti_j^f . Similarly to the ATFM model [9], we use the following monotone binary variables:

$$w_{j,t}^f = \begin{cases} 1, & \text{if time window for flight f in sector } j \text{ is still open at time } t \\ 0, & \text{otherwise} \end{cases}$$

In view of this definition, the "profile" of the decision variables for a specific flight *f* and airspace element *j* over time is as follows:

t	**1**	**2**	**3**	**4**	**5**
$w_{j,t}^f$	1	1	1	0	0

which corresponds to a time window open from time period 1 to time period 3.

Finally, if we suppose that each time window has a minimum (MINTW) and a maximum (MAXTW) width, this hypothesis can be used to fix all the variables prior to time period ti_j^f + MINTW to one and all the variables later than ti_j^f + MAXTW to zero.

B. Notation.

The model's formulation requires the definition of the following notation:

$K \equiv$	set of airports
$S \equiv$	set of sectors
$S^f \subseteq (S \cup K) \equiv$	set of sectors flown by flight *f*, including the origin and destination airports of *f*
$F \equiv$	set of flights
$T \equiv$	set of time periods
$C \equiv$	set of pairs of flights that are continued
$D_k(t) \equiv$	Departure capacity of airport *k* at time period *t*
$A_k(t) \equiv$	arrival capacity of airport *k* at time period

$S_j(t) \equiv$ capacity of sector j at time period t

$s_f \equiv$ turnaround time of an airplane after flight f

$orig_f \equiv$ airport of departure of flight f

$dest_f \equiv$ airport of arrival of flight f

$T_j^f \equiv$ set of feasible time periods for flight f at airspace element j

$l_{jj'}^f \equiv$ minimum acceptable flight time between sector j and sector j' for flight f

$u_{jj'}^f \equiv$ maximum acceptable flight time between sector j and sector j' for flight f

C. Objective function.

The objective function of the model herein presented is the total width of the time windows, to be maximized. In formula:

$$Max \sum_{f \in F, j \in S^f, t \in T_j^f} \alpha_{j,t}^f \cdot w_{j,t}^f$$

Note that, if the objective function's coefficients $\alpha_{j,t}^f$ are all equal to 1, the objective function stands for the total number of time periods assigned to time windows. In fact, by definition of the decision variables, the sum of the variables corresponds to the width of a time window. However, considering sub-linear cost coefficients we are able to include a notion of fairness in our model. In fact, the use of sub-linear cost coefficients helps the distribution of time windows among a larger pool of flights. By distribution of the time windows, we mean that the model will restrain the solution from assigning a large time window to a flight and a small one to the other, if the solutions are equally good. More specifically, the $\alpha_{j,t}^f$ coefficients are defined as follows:

$$\alpha_{j,t}^f = \frac{(t - ti_j^f)^{1-\varepsilon}}{t - ti_j^f} \qquad t \geq ti_j^f + 1$$

being ε a small number greater than zero.

D. Constraints.

The model's constraints set is as follows:

$$\sum_{f \in F: orig_f = k, t \in T_k^f} w_{k,t}^f \leq D_k(t) \qquad \forall k \in K, t \in T \qquad (1)$$

$$\sum_{f \in F: dest_f = k, t \in T_k^f} w_{k,t}^f \leq A_k(t) \qquad \forall k \in K, t \in T \qquad (2)$$

$$\sum_{f \in F: j \in S^f, t \in T_j^f} w_{j,t}^f \leq S_j(t) \qquad \forall j \in S, t \in T \qquad (3)$$

$$w_{j,t}^f \leq w_{j',t+l_{jj'}^f}^f \qquad \forall f \in F, j \in S^f, t \in T_j^f \qquad (4)$$

$$w_{j,t}^f \geq w_{j',t+u_{jj'}^f}^f \qquad \forall f \in F, j \in S^f, t \in T_j^f \qquad (5)$$

$$w_{orig_{f'},t}^{f'} \geq w_{dest_f, t-s_f}^f \qquad \forall (f, f') \in C, t \in T_{orig_{f'}}^{f'} \qquad (6)$$

$$w_{j,t}^f \geq w_{j,t+1}^f \qquad \forall f \in F, j \in S^f, t \in T_j^f \qquad (7)$$

$$w_{j,t}^f \in \{0,1\} \qquad \forall f \in F, j \in S^f, t \in T_j^f \qquad (8)$$

Constraints (1), (2) and (3) define the departure, arrival and sector capacity limits, i.e., they impose that the number of flights which may depart from (or arrive at) airport k, or enter sector j at time period t will not exceed the departure, arrival or sector capacity for the given period. If a time window of a flight f is open at time period t ($w_{j,t}^f = 1$) then one unit of capacity is used (reserved to flight f). This modeling approach ensures the required capacity resources to each flight for the execution of its en-route operations, during any time instant of the assigned time window.

Constraints (4), (5) and (6) impose the consistency of the time windows. Constraints (4) and (5) prevent a time window from being too wide if the time window at the subsequent or previous sector is narrow, respectively. On the other hand, constraints (6) are implemented to guarantee the consistency between time windows of continued flights. Continued flights are those flights for which the aircraft of flight f will be used for a following flight f', with s_f being the minimum amount of time needed to prepare flight f' for departure, following the landing of flight f. Therefore, constraints (6) link the time window of the preceding flight to the time window of the subsequent one.

Finally, constraints (7) define the time connectivity of the model, with the decision variables $w_{j,t}^f$ being monotonic decreasing.

E. Capacity constraints

The mathematical model described above may lead to too conservative solutions since it can declare a large number of flights as critical due to an excessive use of capacity resources. In fact, for each flight and for any period of the time window, the model guarantees all the capacity resources necessary for its execution. For instance, consider the following example: suppose that a 3-period time window is assigned to flight f. Then, as specified above, for each time period, one unit of capacity will be reserved to flight f for a total of 3 units of capacity, even though it will use only one unit. We refer to this approach as "conservative". In what follows, we modify capacity constraints (1), (2) and (3) in order to overcome this issue and we define a capacity "utilization" coefficient for each period of the time window. This capacity utilization coefficient can be any value in the range [0, 1], where value 0 denotes the situation that no capacity is reserved for the execution of the flight operation. For instance, we may consider the case of a capacity utilization coefficient equal to the reciprocal of the width of the time window ("proportional" approach). That is, if we have a 5-period time window, for each period of the time window we are going to reserve only $1/5$ of the capacity. With this purpose, we define new capacity constraint coefficients $\beta_{t,\tau}$ with $\tau \geq t$, as follows:

$$\beta_{t,\tau} = \begin{cases} \dfrac{1}{t+1-ti_j^f} & \text{if } \tau = t \\ \beta_{\tau,\tau} - \beta_{\tau-1,\tau-1} & \text{if } \tau \geq t+1 \end{cases}$$

where the first index refers to the constraint and the second one refers to the variable. In view of the constraint coefficients defined above, the sector capacity constraints (3) become:

$$\sum_{f \in F: j \in S^f, t \in T_j^f} \left(\sum_{\tau \in T_j^f : \tau \geq t} \beta_{t,\tau} \cdot w_{j,\tau}^f \right) \leq S_j(t) \quad \forall j \in S, t \in T \qquad (9)$$

The departure and arrival capacity constraints (1) and (2) can be analogously modified. In Table I, we report an example of the $\beta_{t,\tau}$ coefficients for MAXTW = 5.

For the sake of clarity, we here explain $\beta_{t,\tau}$ coefficients more in detail. Consider the capacity constraint for sector *j* and time period $t = ti_j^f + 1$. The constraint's coefficients for flight *f* are given by the second row of Table I.

TABLE I. EXAMPLE OF $\beta_{t,\tau}$ COEFFICIENTS FOR MAXTW = 5 (PROPORTIONAL APPROACH)

τ - t	0	1	2	3	4
$t = ti_j^f$	1	-1/2	-1/6	-1/12	-1/20
$t = ti_j^f + 1$		1/2	-1/6	-1/12	-1/20
$t = ti_j^f + 2$			1/3	-1/12	-1/20
$t = ti_j^f + 3$				1/4	-1/20
$t = ti_j^f + 4$					1/5

If the time window is still open at time period $t = ti_j^f + 1$ (i.e., $w_{j,ti_j^f+1}^f = 1$), the width of the time window is at least two time periods, and the capacity utilization for flight f is at most an half (thus explaining the coefficient $\beta_{ti_j^f+1, ti_j^f+1} = 1/2$). But, if the time window is also open at time period t+1 (i.e., $w_{j,ti_j^f+2}^f = 1$), its width is at least three, and the capacity utilization for flight *f* is at most one third. Therefore we have to subtract to one half one sixth to obtain one third; and so on.

These modified capacity constraints impose the satisfaction of capacity limits in aggregate terms and they can be considered as a surrogate relaxation of constraints (3). Therefore, at a specific time period *t* it may happen that the actual demand, i.e., the demand that really appears at time period *t*, exceeds the available capacity. This is possible because the number of time windows which include time period *t* can be larger than the available capacity. Under these circumstances, capacity shortages should be addressed at a more operational level by assigning delays.

Obviously, other capacity utilization criteria could be proposed in addition to the two approaches defined so far. For instance, we may consider a third approach ("intermediate" approach) which guarantees for the first time period of the time window the required capacity (one unit) while for the remaining time periods it employs a capacity utilization inversely proportional to the width of the time window. For this approach the capacity constraint coefficients are defined as follows:

$$\beta_{t,\tau} = \begin{cases} 1 & \text{if } t = \tau = ti_j^f \\ 0 & \text{if } t = ti_j^f \wedge \tau \geq ti_j^f + 1 \\ \dfrac{1}{t - ti_j^f} & \text{if } t = \tau > ti_j^f \\ \beta_{\tau,\tau} - \beta_{\tau-1,\tau-1} & \text{if } \tau \geq t+1 \end{cases}$$

In Table II, the values for the $\beta_{t,\tau}$ coefficients for the intermediate case with MAXTW = 5 can be found.

TABLE II. EXAMPLE OF $\beta_{t,\tau}$ COEFFICIENTS FOR MAXTW = 5 (INTERMEDIATE APPROACH)

τ - t	0	1	2	3	4
$t = ti_j^f$	1	0	0	0	0
$t = ti_j^f + 1$		1	-1/2	-1/6	-1/12
$t = ti_j^f + 2$			1/2	-1/6	-1/12
$t = ti_j^f + 3$				1/3	-1/12
$t = ti_j^f + 4$					1/4

F. Ex Post Analysis on the Feasibility of the Solution

Once all time windows for all flights are identified, each flight is entitled to use any of the available time units within a time window to execute the corresponding operation. If the capacity utilization is computed in accordance with the conservative approach, all capacity constraints are certainly respected, regardless of the actual time units taken by flights. However, this might not be true under the proportional and intermediate approaches, as shown by the following example.

Let us calculate the capacity utilization with the proportional approach. Let flight *A* have time window [10,11] at the departing airport *D* and time window [13,14] at sector *J*. Let the time needed by flight *A* for flying from *D* to *J* be equal to three time units. Moreover, let another flight *B* have the time window [7,8] at a different departing airport *D'* and time window [13,14] at the same sector *J*. Let the time needed to flight *B* for flying from *D'* to *J* be equal to six time units. Finally, let the capacity at sector *J* be equal to one in both time units 13 and 14 (see Fig. 1). If flight *A* departs at time 10 and flight *B* at time 8 (or flight A at time 11 and flight B at time 7) the capacity constraints in *J* are respected. But if flight A takes off at time 10 and flight B at time 7(or flight A at 11 and flight B at 8) the mathematical capacity constraints (9) are respected (since $\beta_{t,\tau}$ = ½), but the actual capacity constraints are not.

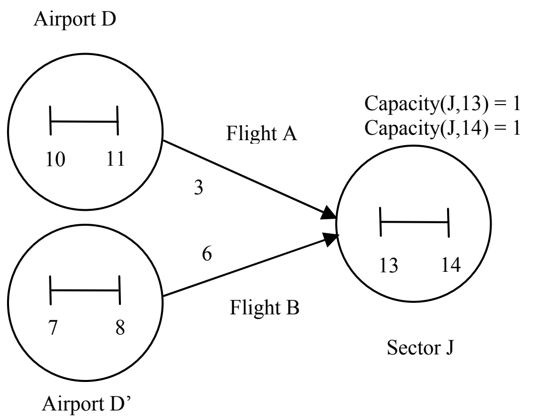

Figure 1. Possible capacity constraint violation

Thus the proportional and intermediate approaches may propose solutions which may not be feasible in practice. In the last part of our study, we analyze to what extent this phenomenon may occur. Once all time windows are given, for each flight we randomly choose the departure time within the available time units of the time window at the departure airport. From this departure time we calculate the entrance time at each sector of the flight route and the arrival time at the destination airport. Thus for each pair (airport/sector, time unit) we are in the position to identify whether its capacity is actually respected or not.

To evaluate the effects of different choices, the random departure times are taken in accordance with three probability distribution functions.

– *Uniform*. All time units of a time window can be chosen with equal probability

– *Triangular*. The probability monotonically decreases with time. Hence the first time unit has the highest probability and the last time unit the lowest one.

– *Step function*. The initial time unit has probability ½ to be chosen, whereas all the other time units equally share the remaining probability.

III. COMPUTATIONAL EXPERIENCE

Our computational experiments on the mathematical model described above were conducted on a series of randomly generated instances whose dimension is comparable to the largest-size cases that can be encountered in practice, i.e., these instances are representative of problems of national scope in the US or near-continental scope in Europe. More specifically, we generate instances which consider 30 airports, 10 of which are hubs, 145 sectors, 50 time periods, and a number of flights involved equal to 6475 flights. The instances are grouped in several sets, each simulating a different level of congestion. To enforce sectors congestion, we suppose a capacity reduction of some sectors. The reduction of capacity affects 3 sectors at a time, for 5 consecutive time periods. Afterwards, three contiguous sectors experience capacity reduction for other 5 consecutive time periods, and so on. In this way, we "simulate" the effect of a bad weather front which moves along a certain direction [9].

For these experiments, the minimum time window size was set to 5 minutes, while the maximum time window size was set to 15 minutes. All the 3 different approaches for considering the capacity have been evaluated, and the results will be presented for all the different cases.

The mathematical model has been implemented in Mosel modeling language, using the Xpress IVE programming environment. The computational times to compute optimal solutions of the model are very short. On average it requires less than 40 seconds and it rarely exceeds one minute of computation on a standard laptop. The good computational performances are related to the structure of the model, which can be cast as a special type of multi-knapsack problem with side constraints.

We here summarize the findings of our computational experience. We define four groups of flights. For each flight of the first group (depicted in the following diagrams by a line with diamond indicators) the size of all the time windows is equal to five minutes. We refer to these flights as "most critical". A second group (line with square indicators) of flights has all the time windows 10-minute large, and a third group (line with triangle indicators) where all flights have only 15-minute time windows, thus representing the least critical flights. Finally there is a fourth group of flights, whose time window widths are not constant through the different sectors of its trajectory from the origin to the destination airports. In the following we refer to these flights as "heterogeneous" as only certain flight operations are critical, those with a 5-minute time window, while for other operations a slack time is allowed, as denoted by larger time windows. For instance consider the case where a flight has a 5-minute time window to fly through a sector, say sector *j*. This flight has some flexibility regarding its time of departure but it has to adjust its speed accordingly in order to meet the narrow time window at sector *j*.

A. Results on capacity utlization approaches

We compare the three criteria presented in Section II-E for the capacity utilization, that is, the conservative, the intermediate and the proportional approach. For each approach, we report a diagram (Figures 2 to 4) displaying the flights' distribution among the four groups of flights under different levels of congestion. The level of congestion in the airspace is reported on the abscissa as the percentage of the available capacity with respect to the nominal capacity. Hence the value 10% represents the case with only 10% of the nominal capacity available (i.e., a situation of maximum congestion) and the value 100% corresponds to the minimum congestion level as all the capacity is available. As expected, the number of most critical flights decreases with the lowering of air traffic congestion.

The percentage of most critical and heterogeneous flights shows a wide variability across the capacity utilization criteria. In fact, for the less congested instances, the percentage of most critical flights varies from 0% (proportional approach, Fig. 3), up to 17% (conservative approach, Fig. 2). Moreover, under the conservative approach the heterogeneous flights represent the largest group. Summing the percentage of most critical and heterogeneous flights, we obtain that almost half of the flights have at least one critical flight phase in all the instances solved, even those with a low level of congestion. These statistics support our observation on the conservative nature of solutions computed by using the first approach. On the other side, the proportional capacity utilization approach seems quite inattentive in guaranteeing the required capacity resources for all the flight operations: the percentage of both the most critical and heterogeneous flights is almost irrelevant, especially for instances with a low level of congestion.

A compromise solution is given by the intermediate approach (Fig. 4) where the sum of the percentages of most critical and heterogeneous flights always lies between 20% and 30%.

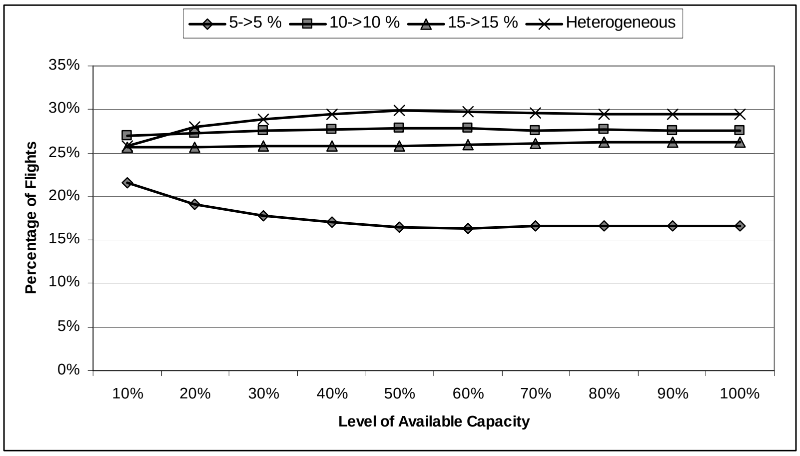

Figure 2. Distribution of flights by their criticality with capacity reduction – conservative approach

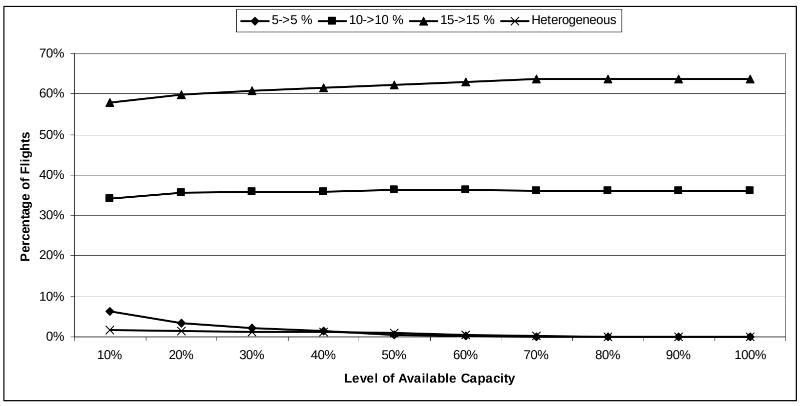

Figure 3. Distribution of flights by their criticality with capacity reduction – proportional approach

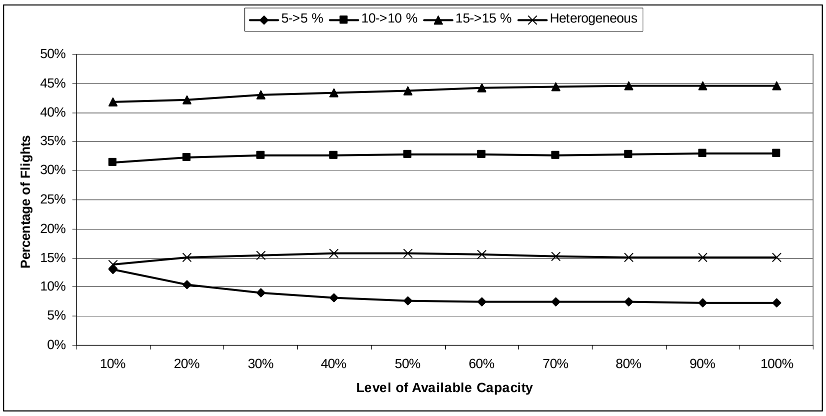

Figure 4. Distribution of flights by their criticality with capacity reduction – intermediate approach

B. *Results on ex post feasibilty analysis*

The application of the proportional and (to a lesser extent) intermediate approaches might lead to capacity shortages that should be addressed at a more operational level by assigning delays. This calls for an evaluation on the robustness of solutions and a quantification of the additional delays that have to be assigned in order to satisfy capacity constraints.

The ex post feasibility analysis (see Section II-F) identifies at each time unit whether sector capacities are violated or not. We consider 1200 random instances per level of congestion, per probability distribution, and per capacity utilization approach. As the test instances have about 50 time units and 200 airports/sectors, we consider approximately 10000 pairs (sector, time).

Tables III and IV show the main results of the feasibility analysis under the proportional and intermediate approach, respectively. The first column indicates the level of available capacity in the airspace following the same rationale of Figures 2 to 4.. The figures in the other columns indicate how many pairs (sector, time) out of 10000 have the capacity violated under the corresponding probability distribution function (see Section II-F) for setting the random departure times. These figures are the average values over the 1200 random instances performed during the simulation exercise.

TABLE III. FEASIBILITY ANALYSIS

PROPORTIONAL APPROACH

Capacity	Uniform	Triangular	Step
10%	14.1	10.3	11.9
20%	10.5	8.7	9.7
30%	7.1	6.1	6.8
40%	5.7	5.2	5.6
50%	5.4	5.2	5.4
60%	4.3	4.1	4.3
70%	3.6	3.4	3.7
80%	2.7	2.8	3.1
90%	2.4	2.4	2.6
100%	2.3	2.3	2.5

TABLE IV. FEASIBILITY ANALYSIS

INTERMEDIATE APPROACH

Capacity	Uniform	Triangular	Step
10%	4.1	1.7	3.4
20%	2.5	0.9	2.4
30%	2.2	0.7	2.1
40%	2.0	0.5	1.9
50%	1.7	0.3	1.6
60%	1.8	0.4	1.7
70%	1.8	0.4	1.7
80%	1.7	0.3	1.6
90%	1.5	0.3	1.5
100%	1.5	0.3	1.4

The results show that for each level of congestion and each probability distribution function the number of capacity-violated pairs is on average always higher under the proportional approach rather than the intermediate approach. These findings are not surprising because the proportional approach allows more flights than the intermediate approach to enter a given sector in a unit of time. Nevertheless, the absolute values of the capacity violations are always extremely low also under the proportional approach. In fact, the highest number of pairs (sector, time) whose capacity is not respected occurs in the case of maximum congestion when the departure time units are chosen in accordance to the uniform distribution. But this value is equal to 14.1 out of 10000, i.e., 0.141%.

The feasibility analysis demonstrates that also in the case that guarantees the maximum level of flexibility to flights (under the proportional approach the number of most critical flights is almost negligible when the level of congestion is low – see Fig. 3) it is a very rare circumstance that the capacity of a sector is not respected. Then we may conclude that the framework proposed in this study not only identifies which flights are critical (all their operations should be executed on time), but also indicates to non-critical flights a degree of temporal flexibility to perform their operations. This flexibility granted to flights has a negligible impact on the airport/sector capacity utilization also in very congested situations, and thus does not produce any significant additional ATC delay.

IV. Conclusions

Latest figures show that in more than a quarter of cases the cause of a flight cancellation or delay is due to circumstances within the airline's control. The effects of such undesired events are not evenly spread among flights because an 'airline' delay may lead on some flights to an additional delay imposed by the ATC authorities with a further increment of delays, depending on the congestion level of the airspace these flights have to fly through. To support an airline operator in understanding which of its flights require specific attention from both the ATC authorities and the airline itself, this paper develops a mathematical model that identifies critical flights.

The model is based on the definition of a set of time windows of variable size for each flight. A time window is a period of time associated with a specific phase of the flight, e.g., taking off, landing and entering sectors, which has to be executed within it. The width of the time window delineates the degree of flexibility granted to flights to carry out a specific operation.

Even though a time window associated to a given flight and airspace resource can be composed by more than one time unit, the flight will actually perform the operation (i.e., occupy the resource) in only one of the available time unit. However, the proposed model is quite flexible and it allows to formulate different approaches for the occupancy of resources. Herein, we have considered three different ways of computing the occupation of the resources (or the utilization of the capacities). The first option is that the resource is fully occupied by the flight in all the time units of the time window (conservative approach). In the second option for each time unit the level of utilization of the resource capacity is inversely proportional to the size of the time window (proportional approach). The final option proposes that the flight uses one full unit of the resource capacity only in the initial time instant of the time window, and a capacity level inversely proportional to the time window size in the remaining time instants (intermediate approach).

Our results show that the conservative approach declares a large number of critical flights due to an excessive use of capacity resources. On the opposite side, the proportional approach identifies a very limited number of critical flights, as no time unit of the time window is really guaranteed for usage. A more balanced solution follows from the intermediate approach as each flight has always granted the initial time unit of its time windows, but not the remaining time units.

It follows that there is the need to evaluate to what extent sector capacities are respected under the proportional and intermediate approaches. Given the set of all time windows for all flights, we simulate the airspace resource occupancy of each sector by randomly choosing the departure time of a flight within the available time units of its departure time window. In this way we determine which time unit of its subsequent time windows the flight will use, and, considering all flights, whether the sector capacity is violated or not in each time unit. Our findings show that on average sector capacities are respected with a rate always higher than 99.8% also when severe congestion occurs, thus proving the viability of the proposed framework to determine the degree of freedom that can be given to flights without degrading the performances of the ATM system.

Further developments of this work may include a) a more realistic description of the air traffic flow by generating random instances showing the traditional peaks and valleys of demand throughout the day, b) an analysis of the interaction among flights in terms of impact on time windows' positions and widths when flights do not respect (some of) their time windows, and c) an analysis of the spatial distribution of the time windows' widths: airports or sectors with a large number of small time windows would be identified as critical resources or bottlenecks for the system.

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