Short-term allocation of Time Windows to flights through a distributed market-based CDM mechanism

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In the case of multiple capacity constrained air traffic resources we propose an Individual Rational and weakly Budget Balanced market mechanism which allows flights to pay for reducing their delays or get compensations if they accept an increased delay with respect to the First-Planned-First-Served rule. To each flight we associate a set of Time Windows, i.e., a sequence of time periods through which flights execute their Business Trajectories. We derive Time Window prices and their corresponding allocation by means of a primal heuristic using a distributed approach based on the Lagrangian relaxation. Some computational experience based on a real case instance is reported.

I. Introduction

The Single European Sky ATM Research Programme (SESAR) identifies the technological steps and the modernization priorities necessary for implementing the new European Air Traffic Management (ATM) through a reorganization of systems, organizations and responsibilities of the different air traffic stakeholders.[1] One of the key concepts in SESAR is the notion of Business Trajectory, owned by the airspace user, which evolves through several phases and represents the best trade-off between ATM constraints imposed by infrastructural and environmental restrictions and the users internal business objectives. Business Trajectories will be expressed in 4 dimensions (latitude, longitude, flight-level and time).

A possible mechanism to formalize the Business Trajectory comes from the concept of Contract of Objectives (CoO) which has been developed by the Contract-based Air Transportation System (CATS) research project (www.cats-fp6.aero). The CoO is a formal and collaborative commitment of ATM actors, i.e., airspace users, airports and Air Navigation Service Providers (ANSPs), to the conduction of each flight. It establishes a sequence of spatial and temporal constraints which constitute milestones to be met during the flight execution. These 4D intervals are called Target Windows and are agreed upon all involved actors for specific transfer of responsibility areas (e.g. between sectors). They represent the commitment to deliver a particular aircraft inside such temporal and spatial intervals. In other words, the proposed CoO consists of a collection of target windows defined at each area where responsibility between actors is transferred and the Business Trajectory should then go through these different target windows.

When an unexpected imbalance between capacity and demand is detected on a short notice, SESAR states that airspace users (i.e., airlines in our context) will be offered the possibility to indicate to the Network Management a priority order for flights affected by delays under the so called User Driven Prioritisation Process (UDPP). Since different airlines are in general competitors, market-based mechanisms seem natural ways of implementing the UDPP.

In this context, we propose a market-based negotiation scheme between flights and the Network Management to allocate target windows. The focus is on the 'critical' target windows which are those associated to the scarce air traffic resources, i.e., where the demand exceeds the available capacity. In fact, all the

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remaining target windows of each flight will be accordingly adjusted. For sake of simplicity, in the remainder of the paper we only consider the temporal dimension of the critical target windows, thus in the following we simply refer to them as Time Windows (TWs). Our work also relies on an extension of the current European Air Traffic Flow Management(ATFM) system, under which just one departure ATFM slot may be allocated to a flight when the capacity does not meet the demand. Rather we assume that one TW is explicitly allocated to a flight for each congested sector crossed. In particular, TWs are first allocated to flights following a First-Planned-First-Served rule at no cost, similarly to the current way of allocating ATFM slots. Then a market mechanism is initiated among flights and the Network Management.

Some papers have already proposed market mechanisms to assign limited resources in the context of air traffic. They are usually based on combinatorial auctions for the long-term strategic allocation of airport slots in the US.[2–4] However, an auction mechanism for air traffic resources is not in general appropriate, since some airlines should pay to get the same resources that they currently receive at no cost. To overcome this limitation, our mechanism swaps the TWs allocated by means of the FPFS rule thus introducing a combinatorial exchange.[5]

A market mechanism is *Individual Rational* (IR) if it guarantees that every participant will have a payoff equal or higher than the payoff obtained without taking part to it. A mechanism is Budget Balanced (BB) if it does not require an external subsidization to run properly. In particular, it is strongly BB, if the payments collected by all actors sum up to zero (i.e., it just redistributes money among participants) or weakly BB if the payments collected by the Network Management can only be positive or equal to zero.

Our market mechanism is IR and weakly BB. Every airline as well as the Network Management are guaranteed to have a non-negative payoff from the participation. Furthermore, it is distributed meaning that flights do not need to communicate to the Network Management confidential information as their cost of the delay. Finally, our proposed mechanism falls within the Collaborative Decision Making (CDM) framework, as airlines have to continually interact with the Network Management. CDM has been successfully implemented in the U.S. through the Ground Delay Program Enhancement[6] and represents a fundamental innovation to be introduced in Europe in the next years through the SESAR programme.

This paper unfolds as follows. Section II illustrates the problem of allocating TWs to flights. Section III formalizes this problem as a combinatorial exchange, introduces TW prices and discusses potentialities and limitations of the centralized approach. Section IV proposes a distributed market mechanism to determine the optimal TW exchange and the associated TW prices. An heuristic algorithm is presented along with some computational results based on a real case instance. Section V summarizes conclusions.

II. The central allocation problem

Let $\mathcal{F} = \{1, ..., F\}$ be a set of flights and $\mathcal{S} = \{1, ..., S\}$ a set of capacity constrained sectors and airports. Each flight $f \in \mathcal{F}$ is expected to cross a sequence of elements $S_f \subseteq \mathcal{S}$ according to its flight plan, hence it will need to be assigned a TW for each $s \in S_f$. A regulated resource $s \in \mathcal{S}$, with capacity limited to K entries per hour from st_time to end_time, has an associated TW Allocation List $L_s = [1, ..., N_s]$. Each TW $j = [I_j, U_j] \in L_s$ has capacity of one flight where:

$$\begin{aligned} N_s &= \left\lfloor \frac{end\_time - st\_time}{\frac{60}{K}} \right\rfloor \\ I_j &= \left\lfloor st\_time + (j-1) \cdot \frac{60}{K} \right\rfloor \text{ with } j \in \{2, ..., N_s\} \\ U_j &= I_{j+1} - 1 \text{ with } j \in \{1, .., N_s - 1\}, \end{aligned}$$

and $I_1 = st\_time$, $U_N = end\_time$. We assume that the Flight Plan also indicates an estimated time of entry into each element $s \in S_f$ traversed by flight f, i.e. E^s_f. Then f is allocated a list of TWs $q_f = [sl_1, .., sl_{|S_f|}]$, where sl_i is the TW assigned on the i^{th} element of S_f and can not be earlier than E^i_f since flights cannot be anticipated, i.e. $E^i_f \leq U_{sl_i}$ for all $sl_i \in q_f$. Additionally whenever $|S_f| > 1$, we assume that the flying time between pairs of consecutive elements, (i, j) with $i, j \in S_f$ and $j = i + 1$, is fixed. This implies that $|I_{sl_j} - I_{sl_i}| \leq E^j_f - E^i_f \leq |U_{sl_j} - U_{sl_i}|$. An assignment q_f will cause a positive delay to flight f if and only if

$E_f^i < I_{sl_i}$ for some $sl_i \in q_f$ and the amount of delay will be:

$$d_f^{q_f} = \begin{cases} \max_{i \in S_f}(I_{sl_i} - E_f^i) & \text{if } E_f^i \leq I_{sl_i} \quad \forall i \in S_f \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

Hence each assignment of TWs q_f to a flight f implies a nonnegative cost of delay $C(f, q_f) \geq 0$, which depends on several factors, e.g, the type of aircraft, the number of connecting passengers, the time of day, the amount of delay.[7] Possibly, airlines only can correctly estimate the real costs of the delays of their flights, hence they have to be queried in a mechanism that seeks to minimize the cost of a TW allocation.

An assignment q_f is feasible for flight f if and only if (i) it contains one TW for each $i \in S_f$ and each pair (i, j) of consecutive TWs is connected by the fixed flying time $E_f^j - E_f^i$, (ii) it assigns a nonnegative delay to f and (iii) the delay it assigns is bounded, i.e. $d_f^q < MaxDel_f$ where $MaxDel_f$ is a fixed parameter for each flight beyond which the flight prefers to be canceled.

Let us indicate with Q_f the set of all assignments that are feasible for flight f, then the optimal assignment of TWs to flights can be formulated as the following 0-1 IP program:

$$Z_{IP} = \min \sum_{f \in \mathcal{F}} \sum_{q \in Q_f} C(f, q) x(f, q) \tag{2a}$$

$$\text{s.t.} \sum_{f \in \mathcal{F}} \sum_{q \in Q_f : q \ni sl} x(f, q) \leq 1 \quad \forall s \in \mathcal{S}, sl \in L_s \tag{2b}$$

$$\sum_{q \in Q_f} x(f, q) = 1 \quad \forall f \in \mathcal{F} \tag{2c}$$

$$x(f, q) \in \{0, 1\} \quad \forall f \in \mathcal{F}, q \in Q_f. \tag{2d}$$

The objective is to find the minimal cost assignment such that each TW is allocated at most to one flight (constraints (2b)) and each flight is assigned one bundle of TWs among its feasible ones (constraints (2c)). The binary decision variable $x(f, q)$ will be one if flight f receives the q^{th} bundle in its request set Q_f, zero otherwise.A feasible solution will exist if and only if there are enough TWs and requests, such that the assigned bundles are pairwise disjoint, i.e. they do not share any TW. To guarantee the existence of a feasible solution we assume that each regulated resource $s \in \mathcal{S}$ has an infinite capacity after the termination of its regulation and that each flight has a request $q_w \in Q_f$ that includes only TWs after the termination of each regulation traversed. The cost $C(f, q_w)$ associated to this bundle will be equal to either the cost of delay caused by such a bundle or to the cost of cancellation in the case this delay exceeds $MaxDel_f$.

Problem (2) can be reduced to the following standard Weighted Set Packing Problem:

$$Z_{SPP} = \max \sum_{f \in \mathcal{F}} \sum_{q \in Q'_f} (\overline{C} - C(f, q)) x(f, q) \tag{3a}$$

$$\text{s.t.} \sum_{f \in \mathcal{F}} \sum_{q \in Q'_f : q \ni sl} x(f, q) \leq 1 \quad \forall s \in \mathcal{S}', sl \in L_s \tag{3b}$$

$$x(f, q) \in \{0, 1\} \quad \forall f \in \mathcal{F}, q \in Q'_f \tag{3c}$$

where $\overline{C} > |F| \cdot \max_{q \in \cup Q_f} C(f, q)$ is chosen such that the Lagrangian problem obtained from problem (2) by dualizing constraints (2c) gives the same assignment as the original problem. This prevent some flight $f \in \mathcal{F}$ to be excluded from the assignment since the cost induced by its exclusion is not compensated by any other possible assignment. The request sets Q'_f are obtained by appending one dummy TW sl_d^f on an dummy resource d to each bundle q, such that each flight $f \in \mathcal{F}$ has one different dummy TW associated and this TW is included in all its requests $q \in Q'_f$. This prevent multiple assignments to the same flight. Then the new set of resources is $\mathcal{S}' = \mathcal{S} \cup d$.

Problem (3) can be interpreted as a sealed-bid type of combinatorial auction, in which all flights communicate their costs $C(f, q)$ to the auctioneer which centrally solves problem (3) according to this information, to find an assignment $X^* = \{q_1^*, ..., q_F^*\}$ that maximizes the social welfare. Successively the auctioneer computes a vector of nonnegative prices for the bundles $P = \{p(q_1^*), ..., p(q_F^*)\}$ and charges each flight f the price

$p(q_f^*)$ of the bundle assigned. As in standard auction theory, we assume that each flight has a quasi-linear utility $u(f, q_f^*) = -C(f, q_f^*) - p(q_f^*)$. Then the auction is not IR, since $u(f, q_f^*) \leq 0$ for all $f \in \mathcal{F}$.

To force IR we rather propose to calculate the standard $FPFS$ assignment $A = \{a_1, ..., a_F\}$ as in the current system and to consider it as the initial endowment for each flight. This is a feasible solution for problem (3) but not necessarily optimal, then we propose to implement the exchange between A and X^* such that every flight f for which $a_f \neq q_f^*$ pays the price $p(q_f^*)$ for its optimal bundle but also receives the payment $p(a_f)$ for the released bundle a_f. Then each flight is a potential buyer and seller of TWs and its utility after the exchange will be $u(f, e^*) = [C(f, a_f) - C(f, q_f^*)] - [p(q_f^*) - p(a_f)] = -C(f, e_f^*) - p(e_f^*)$

III. Pricing the exchange

In this section, we formulate the optimal exchange problem as the following binary IP model:

$$Z_{IP-E} = \max \sum_{f \in \mathcal{F}} \sum_{q \in Q_f} V(f, q) x(f, q) \tag{4a}$$

$$\text{s.t.} \sum_{f \in \mathcal{F}} \sum_{q \in Q_f : q \ni sl} x(f, q) \leq 1 \quad \forall s \in \mathcal{S}, sl \in L_s \tag{4b}$$

$$\sum_{q \in Q_f} x(f, q) = 1 \quad \forall f \in \mathcal{F} \tag{4c}$$

$$x(f, q) \in \{0, 1\} \quad \forall f \in \mathcal{F}, q \in Q_f \tag{4d}$$

where $V(f, q) = [C(f, a_f) - C(f, q)]$ is the value obtained by flight f by exchanging bundle a_f with bundle q and will be positive if the delay caused by bundle q is lower than delay caused by a_f, negative otherwise. We want to find prices such that for each flight f, $u(f, e^*) \geq 0$ in order to guarantee IR and $\sum_{f \in \mathcal{F}} p(e_f^*) \geq 0$ in order to guarantee (weak) BB.

We define Z_{LP-E} as the objective value of the linear relaxation of problem (4), whose dual problem is:

$$Z_{DLP-E} = \min \sum_{f \in \mathcal{F}} u_f + \sum_{s \in \mathcal{S}'} \sum_{sl \in L_s} p(sl) \tag{5a}$$

$$\text{s.t.} u_f + \sum_{s \in \mathcal{S}'} \sum_{sl \in (L_s \cap q)} p(sl) \geq V(f, q) \quad \forall f \in \mathcal{F}, q \in Q'_f \tag{5b}$$

$$p(sl) \geq 0 \quad \forall s \in \mathcal{S}', sl \in L_s \tag{5c}$$

A competitive equilibrium is a situation in which there is a vector of prices p, usually referred to as market clearing or supporting prices[8] and a feasible allocation $(b_1, ..., b_F)$ such that:

$$\begin{aligned} V(f, b_f) - \sum_{sl \in b_f} p(sl) &> V(f, q_f) - \sum_{sl \in q_f} p(sl) \quad \forall f \in \mathcal{F}, q_f \in Q_f \\ \sum_{s \in \mathcal{S}'} \sum_{sl \in L_s} p(sl) &= \sum_{f \in \mathcal{F}} p(b_f) \end{aligned}$$

The formulation of problem (5) suggests the interpretation of dual variables as supporting prices $p(sl)$ for TWs. Let us assume for the moment a linear structure of prices, i.e. for of each bundle of TWs $q \in Q_f$ its price will be $p(q) = \sum_{sl \in q} p(sl)$.

The complementary slackness conditions for the primal-dual pair are:

$$x^*(f, q) > 0 \Rightarrow u_f^* + \sum_{s \in \mathcal{S}'} \sum_{sl \in (L_s \cap q)} p_{sl}^* = V(f, q) \tag{6a}$$

$$u_f^* + \sum_{s \in \mathcal{S}'} \sum_{sl \in (L_s \cap q)} p(sl) > V(f, q) \Rightarrow x^*(f, q) = 0 \tag{6b}$$

$$\sum_{f\in\mathcal{F}}\sum_{q\in Q_f:q\ni sl} x(f,q) < 1 \quad \Rightarrow p(sl) = 0 \tag{7a}$$

$$p(sl) > 0 \Rightarrow \sum_{f\in\mathcal{F}}\sum_{q\in Q_f:q\ni sl} x(f,q) = 1 \tag{7b}$$

Conditions (6) and (7) imply that a linear pricing rule based on individual prices $p(sl)$ implements an individual rational mechanism for our TW-exchange problem, whenever the LP relaxation of problem (4) gives an integer solution. In fact each flight f will weakly prefer to sell the bundle of TWs $a_f \in Q_f$ received under FPFS allocation if it is not optimal for problem (2a) and to buy the bundle of TWs $q^*_f \in Q_f$ optimal for problem (4), because its utility increases:

$$C(f,a_f) + \sum_{sl\in a_f} p(sl) - C(f,q^*_f) - \sum_{sl\in q^*_f} p(sl) \geq 0 \tag{8}$$

Such a payment rule is (weakly) BB, since it can produce a monetary surplus for the auctioneer. In fact all TWs unassigned under FPFS which are assigned in the optimal allocation have a price $p(sl) > 0$ according to (7b). This price has to be paid by the receiving flight but is not due to anyone since that TW was unallocated under FPFS. On the contrary, according to (7a) any TW that is not assigned in the optimal allocation has a price $p(sl) = 0$, meaning that the flight giving up a TW received under FPFS which is not allocated by the market mechanism does not receive any compensation for it.

Unfortunately, supporting prices induced by conditions (6) and (7) exist only in the case that the integrality gap between problem (4) and its LP relaxation is null. This will always be verified in the case of gross-substitute valuation functions,[9] i.e., if for every pair of price vectors $p' \geq p$ (component-wise comparison), we have that the optimal TW package demanded by a flight f at prices p' contains all the TWs in the optimal package demanded by f at prices p, whose price remained constant. The class of gross-substitutes is a largest set of valuation functions which contains unit-demand ones, i.e. when all flights demand bundles are composed by a single TW. In those situations problem (4) reduces to an assignment problem, thus implying that its linear relaxation gives integer feasible solutions, and dual variables define linear market-clearing prices. Whenever there are complementarities in valuations functions (i.e. in the case of multiple constrained resources), gross-substitutes property does not hold anymore.

An alternative approach to identify supporting prices is based on the Vickrey-Clarke-Groves (VCG) class of auction.[10–12] It is a central scheme in auction theory and mechanism design since it is the only general class of auction mechanisms which verifies the Incentive Compatibility property. A market mechanism is Incentive Compatible (IC) if the equilibrium strategy for participants is to report their preferences truthfully, i.e. if they can never increase their individual payoff by misrepresenting their cost functions and thus they have no incentive to untruthfully report these costs, independently from what is the information communicated by others.

Unfortunately when this VCG pricing rule is adopted in an exchange model, the resulting mechanism is IR and IC but not BB. In fact, the Network Management may experiences a loss and then subsidizes the users. Moreover an impossibility result states that it is impossible to build an exchange mechanism that is at the same time IR, BB and IC.[13] However several VCG-based payment rules that clear combinatorial exchanges while guaranteeing BB and IR and minimizing the distance from VCG prices have been proposed.[5] Experimental and theoretical analysis performed on several distance functions suggests that a "Threshold rule" has useful incentive properties and provides allocative efficiency higher than other rules, by removing easy opportunities for manipulation. In fact, the Threshold rule minimizes the maximal amount that a participant can increase its utility by misrepresenting its information.

IV. A distributed market mechanism

The central allocation problem that determines the optimal exchange, could result complicated to be adopted in practice, notably because (i) it requires the complete disclosure of airline private information regarding their cost and (ii) the computational burden for solving Problem (4) is entirely faced by the Network Management which must solve one NP-hard problem for each allocation (plus other $|F|$ NP-hard problems to calculate Δ_{VCG} in the case VCG prices are charged). In order to avoid this issues we propose in the following a distributed algorithm that exploits the decomposition properties of Problem (4). In fact, by dualizing constraints (2b) the corresponding Lagrangian formulation of problem (4) is:

$$ZLR_{LP-E}(\lambda) = \max \sum_{f\in\mathcal{F}} \sum_{q\in Q_f} V(f,q)x(f,q) + \sum_{s\in\mathcal{S}, sl\in L_s} \lambda_{sl}(1 - \sum_{f\in\mathcal{F}, q\in Q_f : q\ni sl} x(f,q)) \tag{9a}$$

$$\text{s.t.} \sum_{q\in Q_f} x(f,q) = 1 \quad \forall f\in\mathcal{F} \tag{9b}$$

$$x(f,q) \geq 0 \quad \forall f\in\mathcal{F}, q\in Q_f \tag{9c}$$

Problem (9) is separable into F problems, one for each flight and can be solved locally by Aircraft Operators, according to problem (10), which is a linear problem and can thus be solved in polynomial time:

$$ZLR_{LP-E}(f,\lambda) = \max \sum_{q\in Q_f} V(f,q)x(f,q) + \sum_{s\in\mathcal{S}, sl\in L_s} \lambda_{sl}(1 - \sum_{q\in Q_f : q\ni sl} x(f,q)) \tag{10a}$$

$$\text{s.t.} \sum_{q\in Q_f} x(f,q) = 1 \tag{10b}$$

$$x(f,q) \geq 0 \quad \forall q\in Q_f \tag{10c}$$

For each TW sl, its prices $\lambda(sl)$ are calculated centrally according to the excess of demand for it and then communicated to Aircraft Operators, which will in turn modify the demand for TWs according to such prices. The following algorithm can be employed to calculate prices

$$\lambda_{sl}^{k+1} = \max(0, \lambda_{sl}^{k} - S_r^k \cdot SG_s^k) \tag{11a}$$

$$SG_s^k = 1 - \sum_{f\in\mathcal{F}, j\in Q_f : q\ni sl} x(f,j) \tag{11b}$$

where S_r^k is a positive stepsize chosen at iteration k and SG_s^k is a subgradient of $ZLR_{LP-E}(\lambda)$ at any λ for which x solves problem (9). Thus ideally the Network Management seeks the prices λ that solve the following dual problem

$$ZLR_{LP-E} = \min_{\lambda} ZLR_{LP-E}(\lambda) \tag{12a}$$

$$\text{s.t.} \lambda \geq 0 \tag{12b}$$

Since $ZLR_{LP-E}(\lambda)$ is a convex, piecewise linear, non-differentiable function, this problem is typically solved through a subgradient algorithm. The resulting procedure iteratively alternates a central price-calculation phase (problem 11) with a local optimization one which finds the maximal value TW-exchange at current prices (problem 10). By appropriately choosing the stepsize S_r^k such that $S_r^k \to 0$ and $\sum_{i=1}^{k} S_r^k \to \infty$ for $k \to \infty$, the procedure converges to a solution which minimizes $ZLR_{LP-E}(\lambda)$.[14] By duality theory it will follow that $Z_{IP-E} \leq Z_{LP-E} \leq ZLR_{LP-E}(\lambda)$, while $Z_{IP-E} = Z_{LP-E}$ in the case of null gap between the integer program and its linear relaxation (i.e. with gross-substitutability) and $Z_{LP-E} = ZLR_{LP-E} = \min_\lambda ZLR_{LP-E}(\lambda)$ when the subgradient algorithm converges to an optimal solution for problem (12).

However this exchange will be optimal for the original problem if and only if the gap between Z_{IP-E} and its linear relaxation is null, a condition which can only be guaranteed in the case of gross-substitute valuations, for example when all participants compete for TWs on a unique resource. Furthermore, even in the case of gross-substitutability, there is no guarantee of convergence in a finite number of steps. By stopping the procedure when a feasible exchange for the original problem (4) is demanded at current prices, the optimality of the solution will be verified if and only if $\sum_{s\in\mathcal{S}, sl\in L_s} \lambda_{sl}(1 - \sum_{f\in\mathcal{F}, q\in Q_f : q\ni sl} x(f,q)) = 0$.[15] This condition is automatically verified in the case that all bundles are singletons (i.e. the single constrained resource problem) due to the complementary slackness condition, but this is no longer true in the general case. Hence if the procedure stops prematurely it will be $ZLR_{LP-E} > Z_{IP-E}$ and some lagrangian multipliers $\lambda_{sl} \geq 0$ will be higher than minimal prices but the correspondent exchange will still guarantee IR and weak BB. We propose in the following section an heuristic algorithm to implement the distributed exchange that exploits some of these properties to realize a practical mechanism for TW exchanges.

A. A heuristic approach

In order to implement a distributed market mechanism that achieves, in a reasonable amount of time, an exchange which is IR and BB we propose the following heuristic. A formula for S_r^k which has been proven effective in practice is:

$$S_r^k = \frac{\mu_k(ZLR_{LP-E}(\lambda^k) - Z^*_{IP-E})}{\sum_{s\in\mathcal{S},sl\in L_s}(1-\sum_{f\in\mathcal{F},q\in Q_f:q\ni sl} x^k(f,q))^2} \tag{13}$$

where $0 < \mu_k \leq 2$, x^k are the solutions to problem (10) at iteration k according to the vector of TW prices λ^k. Usually the scalar μ_k is taken at its higher values during first iterations and halved whenever $ZLR_{LP-E}(\lambda^k)$ has failed to decrease in a specified number of iterations.[16] In our case the Network Management does not know the exchange values $V(f,q)$ and thus cannot calculate neither $ZLR_{LP-E}(\lambda^k)$ nor Z^*_{IP-E}. We then modify formula (13) in the following sense:

$$S_r^k = \frac{\mu_k(UB\_Z^* - ZLB_{IP-E})}{\sum_{s\in\mathcal{S},sl\in L_s}(1-\sum_{f\in\mathcal{F},q\in Q_f:q\ni sl} x^k(f,q))^2} \tag{14}$$

where UB_Z^* is an upper bound on the optimal value of the exchange which is held constant and ZLB_{IP-E} is a lower bound on the optimal value of the exchange for each instance, which is dynamically adjusted through the course of the distributed mechanism. At iteration k the Network Management can in fact calculate for each bundle j demanded by flight f at current prices λ^k, a lower bound on the exact value $V(f,j)$ for the exchange:

$$LB(f,j) = \sum_{s\in\mathcal{S},sl\in L_s:j\ni sl} \lambda^k_{sl} - \sum_{s\in\mathcal{S},sl\in L_s:a_f\ni sl} \lambda^k_{sl} \tag{15}$$

For all $f \in \mathcal{F}$ and $q \in Q_f$, lower bound can be initialized to $LB(f,q) = 0$ if $d_f^q \leq d_f^{a_f}$ and $LB(f,q) = -\infty$ if $d_f^q > d_f^{a_f}$, since we assume non-negative costs of delay. This implies that each flight would exchange its FPFS assigned bundle a_f with q for a cost greater or equal to zero whenever q causes a shorter delay then a_f or for a negative cost (a taking) whenever q represents a longer delay then a_f. At iteration k the Network Management will calculate the $LB(f,j)$ value according to formula (15) and it will store it in memory if it is higher than the previously calculated one. Also it is possible to update with this same value the $LB(f,b)$ for all the bundles $b \in Q_f$ such that $d_f^b < d_f^j$, since the value of an exchange is a decreasing function of delay and $V(f,b) > V(f,j)$. The Network Management can then solve the following problem:

$$ZLB_{IP-E} = \max \sum_{f\in\mathcal{F}}\sum_{q\in Q_f} LB(f,q)x(f,q) \tag{16a}$$

$$\text{s.t.} \sum_{f\in\mathcal{F}}\sum_{q\in Q_f:q\ni sl} x(f,q) \leq 1 \quad \forall s\in\mathcal{S}, sl\in L_s \tag{16b}$$

$$\sum_{q\in Q_f} x(f,q) = 1 \quad \forall f\in\mathcal{F} \tag{16c}$$

$$x(f,q) \in \{0,1\} \quad \forall f\in\mathcal{F}, q\in Q_f \tag{16d}$$

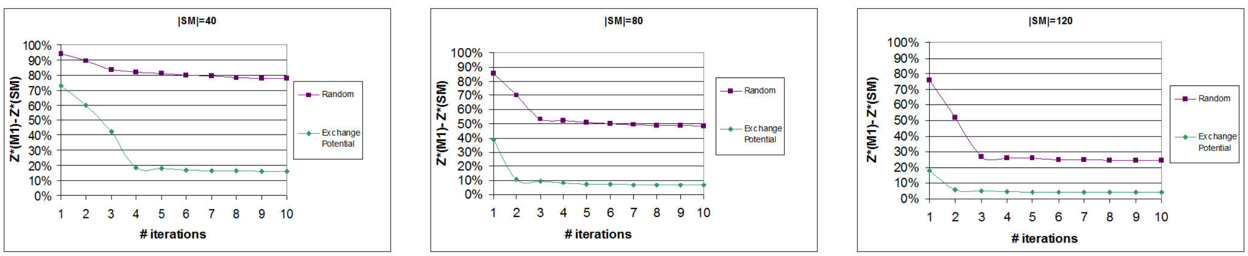
Problem (16) is equivalent to problem (4) with $V(f,q) = LB(f,q)$ for all $f \in \mathcal{F}$ and $q \in Q_f$, then it will be $ZLB_{IP-E} \leq Z_{IP-E}$, the strict inequality holding whenever $LB(f,q) < V(f,q)$ for at least one request $q \in Q_f$ for some flight $f \in \mathcal{F}$. Hence all the integer (feasible) exchanges calculated by solving the linear relaxation of problem (16) with $V(f,q) = LB(f,q)$ and implemented at prices equal to the dual variables corresponding to constraints (16b), will guarantee IR and weak BB, whenever $ZLB_{IP-E} > 0$.

The solution obtained (exchanges and prices associated) is not however a competitive equilibrium because at the given prices there could be some f better-off with another exchange than the one implemented and this implies that the solution must be somewhat forced by the Network Management. Then after a pre-determined number of iterations or a given elapsed time, if an equilibrium cannot be found by simply alternating local optimization and price update (i.e. problems 12 and 11), then the Network Management can impose the best solution calculated so far by solving the linear relaxation of problem (16) at the dual prices, i.e. the integer solution which gives the highest positive-value according to LB, which is the last feasible solution obtained since LB are always updated by increasing them.

The complete procedure for TW exchanges can proceed as it follows. The first step is to create a partition of the grand coalition $\mathcal{F}$ into independent subsets $M_i \subseteq \mathcal{F}$, such that for every pair of different flights $f \in M_i$ and $g \in M_j$ with $i \neq j$ it will be $Q_f \cap Q_g = \emptyset$. Then each subset M_i constitutes an independent market, since all the tradable resources will be within the market itself. From each of these markets, smaller sub-coalitions (sub-markets) of predetermined size Z are formed and then processed, in order to increase the probability of obtaining integer solutions to the linear relaxation of problem (16), thus allowing a computation of exchange and prices in a polynomial time. In fact we have observed experimentally that by reducing the size of the sub-markets, at the same time the value of the optimal exchange reduces while the percentage of instances for which $Z_{IP-E} = Z_{LP-E}$ increases.

Sub-markets are created according to their exchange potential. Flights in Market M_i are first ordered from the one with the lowest to the one with the highest assigned FPFS request. Then starting from the head of this ordered list one flight f is selected as well as its first potential seller g starting from the tail. A flight g is a potential seller for flight f if (i) they share at least one resource s ($S_f \cap S_g \neq \emptyset$) (ii) f prefers the TW k assigned to g on s than its currently assigned one j ($I_k < I_j$) (iii) TW k is feasible for f ($E^s_f \leq U_k$). If no potential seller exists the flight next to f is selected together with its first potential seller. Once a number of flights of the predetermined size Z has been selected, the first sub-market is created. Then a second sub-market which considers only flights which have not been previously included is built. This procedure may iteratively create up to $|M_i|/Z$ distinct sub-markets of size Z.

Relaying on the market M_1 of 425 flights as described in Section B, Figure 1 illustrates the case in which 10 sub-markets of fixed size $|SM|$ are processed in order of their potential of exchange, in comparison with the situation in which they are formed by including flight at random. Each point corresponds to the average on 100 instances of different vectors of costs drawn from the same distribution.



(a) $|SM| = 40$ (b) $|SM| = 80$ (c) $|SM| = 120$

Figure 1. Gap between the optimal value of the exchange in the Main Market $M1$ and in Sub-markets SM.

By forming sub-markets with this procedure the exchanges attainable always give a higher global value than in the case trading occurs within random coalitions. The higher value exchanges are established during the first iterations, i.e. when the flights with higher exchange potential trade, while after 5 iterations only a small number of residual exchanges occur.

The number of instances which give non integer solution to the linear relaxation of problem (4) increases with the size of the sub-market. When $|SM| = 40$ on average 0.15% of cases are non integer, 3.65% when $|SM| = 80$ and 4.35% when $|SM| = 120$. In these cases the dual variables are not supporting prices and one possible solution could be represented by the exchange optimal for the integer problem (4) and the prices calculated according to a VCG-based payment rule. Even if a competitive equilibrium with linear prices does not exist for such instances, our heuristic can still converge to a solution which guarantees IR and weak BB. Once the sub-markets have been formed according to criteria described before, the iterative mechanism can be applied to them.

If after a pre-determined number of iterations $MaxIter$ an equilibrium cannot be found, the last feasible solution calculated with LB is imposed by the Network Management and the correspondent exchange is implemented at the dual prices. The diagram in Figure (2) represents the steps performed by the heuristic procedure.

B. Computational results

We simulated this procedure on a sample of traffic retrieved from real CFMU data relative to the two hours period from 09:00 AM to 11:00 AM on Friday August 15^{th}, 2008. There were a total of 60 capacity

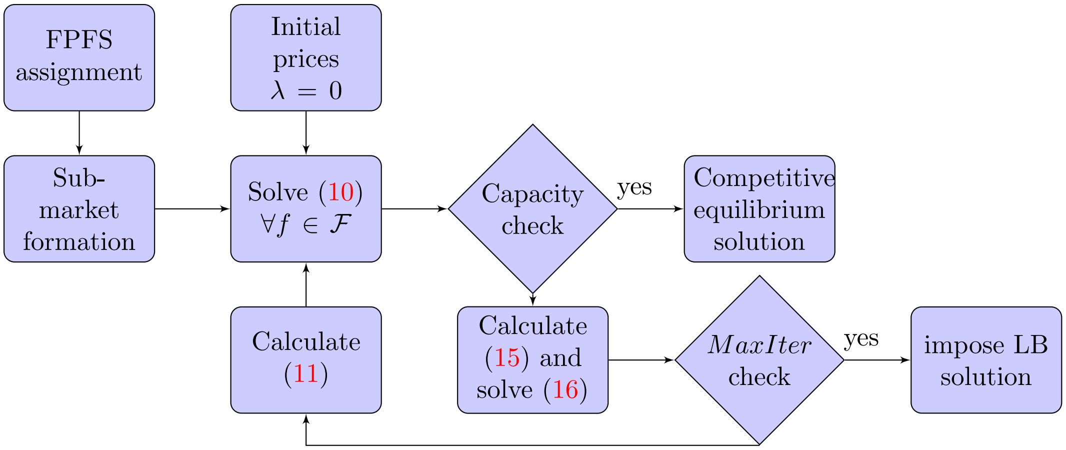


Figure 2. Schematics of the iterative Market Mechanism

constrained resources and 482 regulated flights, that were clustered into 3 independent markets ($M1, ..., M3$), with $|M1| = 425$,$|M2| = 34$,$|M3| = 23$. Market $M1$ included most of the flights interacting directly or indirectly in the exchange of TWs on 58 resources, flights in $M3$ and only them were affected by a regulation on an upper en-route sector (LECMDOM) in the north of Spain that was limiting traffic flow rate to 43 entries/hour from 09:00 AM to 12:00 AM due to ATC capacity reasons. Flights in $M2$ and only them competed for the assignment of TW on an upper en route sector (LFMMW2) located in the South of France that was closed from 10.15 AM to 11.30 AM due to ATC routing. In this such case, i.e. when capacity of a certain resource $z \in \mathcal{S}$ is null for a given time period $[st\_time; end\_time]$, we included just 1 TW in L_z with $I_1 = end\_time$ and infinite capacity in order to make problem feasible without rerouting (the same assumption is used in[17]).

For each flight $f \in \mathcal{F}$ we attached a vector cost of delay $CD_f \subset \mathbb{N}^3$, where each component represents the per-minute cost of delay according to the magnitude of the delay itself, which has been discretized into the three classes $[1; 15)$ min, $[15; 45)$ min, $[45; MaxDel_f]$ min. Components $cd_f \in CD_f$ have been randomly drawn from the uniform distribution on the three discrete intervals $[1; 5]$ €/min, $[15; 25]$ €/min, $[30; 105]$ €/min respectively. Graph in Figure (3) shows the results obtained by applying the iterative Market Mechanism presented schematically in Figure (2).

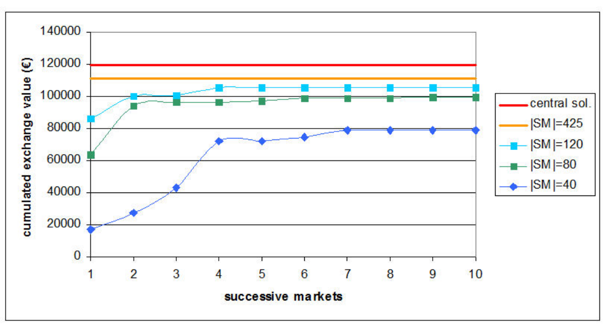


Figure 3. Value of the exchanges obtained through the iterative mechanism on several Sub-Markets

We employed Xpress Mosel v.3.0.0 to code the heuristic procedure and Xpress Optimizer version 20.00.05 to solve all the linear problems. The result figures obtained regarding the cost savings achievable for Aircraft Operators by implementing exchanges are fairly high, especially if we consider that our traffic sample is relative to a 2 hours period. However they represent less than 20% of the total cost of delay originally caused by the FPFS allocation which equals for the traffic sample analyzed 740187 €, according to the algorithm we developed. Obviously it is not the same algorithm employed at CFMU for the allocation of ATFM slots, however this result is perfectly in line with the figures estimated by EUROCONTROL.[18]

V. Conclusions

We introduce the concept of Time Windows as a sequence of time periods through which flights execute their Business Trajectories. If not all airline requests can be accommodated, the Network Management imposes TWs to flights following a FPFS rule, similarly to the current policy of allocating ATFM slots. Furthermore, we assume that airlines are interested in paying for delay reductions or receiving compensations for delay increases and we propose a market mechanism that allows airlines to trade their FPFS-allocated TWs. Our scheme is distributed as a centralized policy based on the airlines' costs cannot be implemented, unless the Network Management knows the delay costs for each flight, which are private information internal to Aircraft Operators. Our mechanism let airlines decide autonomously for each flight whether it is preferable to keep the TW obtained by the FPFS policy or to exchange it at the market price. In the case of multiple capacity constrained resources we show that an Individual Rational and weakly Budget Balanced market mechanism exists. We derive TW prices and their corresponding allocation by means of a primal heuristic using a distributed approach based on the Lagrangian relaxation. We apply our algorithm on a real instance considering approximately 500 flights and 60 sectors. Our results show that the market-based solution allows the participating airlines to decrease their overall ATFM delay-related costs with respect to the FPFS allocation by tens of thousands Euros per day.

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